Contribution of Ancient Jaina Mathematicians

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1. INTRODUCTION

The subject is so wide that volumes can be written on it and hence no single paper can deal with the subject matter comprehensively. Anyhow, in the present paper, starting with a brief history of ancient mathematics, an attempt has been made to touch upon certain aspects of some of the contributions of Jaina mathematicians.[1] It may be noted here that there may be certain controversy regarding the dates and the authority of certain mathematical works, but the facts stated here refer to the standard published works.

Mathematics occupied a very high place in the intellectual life of India in ancient times. In fact mathematics in ancient India was the highest in the world. India was at the top in mathematics in the world uptil the beginning of the 17th century. In northern India, the progress made by Indian mathematicians came to an end in the 12th century, on account of certain historical reasons. In south India, the mathematicians, however, continued the progress up to the beginning of the 17th century. Till then India was leading all the countries of the world in mathematics.

2. IMPORTANCE OF MATHEMATICS IN JAIN RELIGION

Jainas of ancient India attached great importance and took keen interest in the study of mathematics and this subject was regarded as an integral part of their religion. The study of mathematics formed one of the four ***anuyogas*** or auxillary sciences indirectly servicable for the attainment of the solution of soul's liberation known as ***moksa***. ***Ganitanuyoga*** (or the exposition of the principles of mathematics) is one of the four ***anuyogas***, required in the Jainism. The knowledge of ***Samkhyana*** (literally the science of numbers, meaning arithmatic and astronomy) is stated to be one of the principal accomplishments of the Jaina priest.[2] This knowledge was required by him for finding out the proper time and place for the religious ceremonies.[3]

1. About Gaṇita Sara Samgraha of the world fame Jaina mathematician Mahāvīracārya (850 A. D.), see author's paper—
   "On the Ganita Sar Samgraha of Mahāvīra (850 A. D.)" I. J. H. S. 1977.
2. (1) See : Bhagwati Sutra. Sutra 90.
   With the commentary of Abhayadeva Suri (c. 1050).
   Ed. by Agamodaya Samiti of Mahesana, 1919.
   (2) Uttara Dhayana Sutra.
   Eng. Trans. by H. Jacobi, Oxford 1895. Chap. XXV. Sutra 7, 8, 38.
3. See the remarks of Santi Candra Gani (1595 A. D.) in the preface to his commentary on the Jambu Dvipa Prajnapti.

According to Jainas, a child should be taught firstly writing, then arithmetic as most important of the seventy two sciences or arts.[1] According to the Jaina legand, their first tirthankar Rishabnath, taught the Brahmi script to his daughter Brahmi and mathematics to his other daughter Sundari. The sacred literature of the Jainas is called *Siddhanta* or *Agama* and is very ancient. This literature is equally important for their work on Scientific concepts. In fact, Jainas evolved their own theories and made notable contributions to the science of medicine, mathematics, physics, astronomy, Cosmology, the structure of matter and energy and even the atom, the fundamental structure of living beings, the concept of space and time, and the theory of relativity.

Gaṇita sara Samgraha (collection of essence of mathematics) of Mahavira (850 A.D.) is the only treatise on arithmetic and algebra, by a Jaina Scholar, that is available at present. Surya prajnapti and the Chandra prajnapti are two astronomical treatises. The other mathematical treatises by the early Jainas have been lost.

3. AN APPRECIATION OF MATHEMATICS in the words of Mahavira (850 A. D.)

The Indian name for mathematics is *Ganita*. It literally means the science of calculation or computation. The following appreciation of mathematics is given by Mahavira, in his work 'Gaṇita Sara Samgraha' (GSS).

"In all those transactions which relate to wordly, vedic or (other) similar religious affairs, calculation is of use. In the science of love, in the science of wealth, in music and in the drama, in the art of cooking and similarly in medicine and in things like the knowledge of architecture.

In prosody, in poetics and poetry, in logic and grammar and such other things, and in relation to all that constitutes the peculiar value of (all) the (various) arts, the science of computation is held in high esteem.

In relation to the movements of the Sun and other heavenly bodies, in connection with eclipses and the conjuction of planets, and in connection with the *triprasna** and the course the moon—indeed in all these (connections) it is utilised.

The number, the diameter and the perimeter of islands, oceans and mountains ; the extensive dimensions of the rows of habitants and halls belonging to the inhabitants of the (earthly) world of the interspace (between the worlds), of the world of light, and of the world of the gods ; (as also the dimensions of those belonging) to the dwellers in hell and (other) miscellaneous of all sorts—all these are made out by means of computation.

The configuration of living beings therein, the length of their lives, their eight attributes and other similar things, their progress and other such things, their staying together and such other things—all these are dependent upon computation (for their due measurement and comprehension).

What is the good of saying much in vain ? Whatever there is in all the three worlds, which are possessed of moving and non-moving beings—all that indeed cannot exist as apart from measurement.

1. Antagada Dasao and Anuttaro. Vavaya Dasao. Eng. Trans. by L.D. Bernett. 1907, p. 30.

*The triprasana is the name of a chapter in Sanskrit astronomical works, and the fact that it deals with three questions is responsible for that name. The questions dealt with are Dik (direction), Disa (position) and Kala (time) as appertaining to the planets and other heavenly bodies.

With the help of the accomplished holy sages, who are worthy to be worshipped by the lords of the world, and of their disciples and disciples' disciples, who constitute the well known jointed series of preceptors, I glea n from the great ocean of the knowledge of numbers a little of its essence, in the manner in which gems are (picked up) from the sea ; gold is from the stony rock and the pearl from the oyster shell ; and give out, according to the power of my intelligence, the *Sara Samgraha*, a small work on arithmetic, which is (however) not small in value".[1]

The author of the GSS has always held the great Mahavira, the founder of the Jain religion, to have been a great mathematician.[2] Amongst the religious works of the Jainas, that are important from the view point of mathematics are :

(1) Surya Prajnapti (2) Jambu Dvipa Prajnapti	About 500 B. C.
(3) Sthananga Sutra (4) Uttaradhyayana Sutra (5) Bhagwati Sutra (6) Anuyoga-dvara Sutra	About 300 B. C.

4. THERE IMPORTANT SCHOOLS OF MATHEMATICS

In the Sulva Sutra period (750 B. C. to 400 A. D.) there existed three important schools of mathematics :

(1) The Kusumpura or Pataliputra school near modern Patna (latitude 25,37°N, longitude 85.13°E) in Bihar (ancient Magadha) which was a great centre of learning. The famous University of Nalanda was situated in modern Patna, and this was a centre of Jaina scholars in ancient times. Bhadra Bahu (4th Cent. B. C.) and Umaswati (2nd Cent. B. C.) belonged to this school.

(2) The Ujjain School
Brahmagupta (7th Cent. A. D.) and Bhāskaracārya (12th Cent. A. D.) belonged to this school.

(3) The Mysore School
Mahaviracārya (9th Cent. A. D.) or briefly Mahavira belonged to this school.

There was a close contact between the three schools and the mathematicians of one school visited the other schools frequently.

4. 1. KUSUMPURA SCHOOL OF MATHEMATICS

The culture of mathematics and astronomy in the Kusumpura school survived upto the end of the 5th Century of the christian era when flourished the famous algebraist Aryabhata (476 A. D.) who made many innovations in Hindu astronomy. Aryabhata was the Kulpati of the University of Nalanda. He was unanimously acknowledged by the later indian mathematicians as father of the Hindu algebra.

India's first scientific satellite launched on 19th April 1975 at 1 P. M. (I. S. T.) from Moscow is named after this great Indian astronomer and mathematician. India celebrated Aryabhata's 1500th birth anniversary in November 1976 at Indian National Science Academy New Delhi, where many leading

1. See G. S. S. Slokas 9—19. p. 2-3.
2. Compare Chapter 1-2.

mathematicians of the world participated in the deliberations. On this occasion a critical edition of Aryabhata's remarkable work 'Aryabhatiya', with associated commentaries as edited by Dr. Kirpa Shankar Shukla of Lucknow, was released.

The influence of this school continued unabated for several centuries after Aryabhata.[1]

BHADRA BAHU

Bhadrabahu came down from Bihar (Magadha) in 4th century B. C. and settled down at Sarvana Belgola in the Mysore state. On his way he passed through Ujjain and halted there for some time. He was one of the great preceptors of the Jainas and at the same time an astronomer and a mathematician too. He could reproduce from memory the entire canonical literature of the Jainas and was befittingly called a Srutakevalin. Bhadrabahu is the author of two astronomical works :

1. A commentary of the Surya Prajnapti (500 B. C.) ,[2] and
2. An original work called the Bhadra bahavi Samihita.[3]

UMASWATI

Umaswati was a reputed Jaina metaphysician. According to Swetambar Jains, he was born at a place called *Nyagrodhika* and lived in the city of Kusumpura in about 150 B. C. According to this sect, his name is said to be a combination of the names of his parents, the father Swati and the mother Uma. But Digambar Jains' version is that his name was ***Umaswami*** and not ***Umaswati*** and that he lived in the years 135 A. D.—219 A. D. In the present paper Swetambar Jains' version is taken as accepted. The earliest commentator of Umaswati is Siddhasena Gani or Divakara who lived in 56 B. C.

Tattvartha-dhigama—Sutra-Bhāshya is an important work of Umaswati. In this text, an attempt has been made to explain the nature of things and the authority of this work is acknowledged both by the Swetambaras and the Digambaras. Umaswati was also the author of another work known as *Ksetra Samasa* ("Collection of places"). This work is also known as *Jambudvipa samasa*. This work deals with geography and mensuration. It may be noted that *Ksetra samasa* and *Karana bhavana* are two classes of works that give in a nutshell the mathematical calculations employed in Jaina canonical works. The earliest *Kestra samasa* was by Umaswati. It is noteworthy that Umaswati was not a mathematician. The mathematical results and formulae as quoted in his work, it seems, were taken from some treatise on mathematics known at his time.

5. TOPICS IN MATHEMATICS

According to the Sthanaga Sutra,[4] (before 300 B. C.) the topics of discussion in mathematics (Sankhyana or the "Science of Numbers") are ten in number :

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1. At Kusumpura there was another astronomer and mathematician of the name of Aryabhata who was anterior to the Aryabhata of 476 A. D.
   See : "Two Aryabhatas of Al-Biruni".
   Bull. Cal. Math. Sc. Vol. XVII, 1926, p. 68.
2. Sutra 11. Commentary on Surya Prajnapti by Malayagiri (c. 1150).
3. This work was found by Buhler.
   See report on Sanskrit manuscripts 1874—1875 A. D. p. 20.
   About this work it has not been established that it belonged to the Bhadrabahu in question.
4. See Sutra 747.
   "Parikammam vavaharo rajju rasi Kalasavamme ya
   Javantavati vaggo ghano tataka vaggavaggo vikappo ta".

1.	Parikarma	("fundamental operations")
2.	Vyavahara	("subjects of treatment")
3.	Rajju	("rope" meaning "geometry")
4.	Rasi	("heap" meaning "mensuration of solid bodies")
5.	Kala Savarnama	("fraction")
6.	Yavat-tavat	("as many as" meaning "simple equations")
7.	Varga	("square" meaning "quadratic equations")
8.	Ghana	("cube" meaning "cubic equations")
9.	Varga-varga	(biquadratic equations")
10.	Vikalpa or bhong	("permutations and combinations")

The exact meaning of some of the above terms is not known and this has been a subject matter of controversy for the mathematicians. However, in the light of the available text and the usage of the above terms in later Hindu mathematics, we can define the above terms as below :

Parikarma means the four fundamental operations of arithmetic *viz.* addition, subtraction, multiplication and division. *Vyavahara* means applied arithmetic. It is the application of arithmetic to concrete problems. *Kalasavarnama* refers to operations with fractions. Mahavira (850 A. D.) has used these three terms in exactly this sense in his GSS. The first two terms appear indeed in the works of all the Hindu mathematicians from Brahmagupta (7th Cent. A. D.) onwards.

Rajju is the ancient Hindu name for geometry. It was called Sulva in the vedic period. *Rassi* means a heap in general and it may refer to the section on the treatment of the mensuration of solid bodies.

Yavat-tawat is the symbol for an unknown quantity in Hindu algebra. According to Abhayadeva Suri (11th cent. A. D.), the commentator of the Sthananga Sutra (before 300 B. C.) this term refers to multiplication or to the summation of series (samkalita). But obviously multiplication is included in the fundamental operations.

Varga means both square and square-root, and it refers to quadratic equations. *Ghana* means both cube and cube-root, and it refers to cubic equations. *Varga-varga* refers to biquadratic equations. It may be noted here that Abhayadeva Suri (11th cent. A.D.) thought that varga, ghana, varga-varga refer respectively to the rules for finding out the square, cube and fourth power of a number. But in Hindu mathematics from earliest times, these operations were regarded as fundamental operations and hence they are coverd under the first term viz. *Parikarma*. Thus the inference of Abhayadeva Suri is not correct.

Vikalpa or bhong is the Jaina name for permutations and combinations. This topic has been accorded a separate mention on account of its importance in mathematics.

6. MULTIPLICATION AND DIVISION BY FACTORS

In the Tattvartha dhigama-sutra-Bhāshya[1] of Umaswati (150 B.C.), a reference has been made of two methods of multiplication and division. In one method, the respective operations are carried on with the two numbers considered as a whole. In the second method, the operations are carried on in successive stages by the factors, one after another, of the multiplier and the divisor. The former method is our ordinary method, and the later is a shorter and a simpler one. The method of multiplication by factors has been mentioned by

1. See Chap. 11, p. 52.

all the Indian mathematicians from Brahmgupta[1] (7th cent. A.D.) onwards. The division by factors is found in *Trisatika*[2] of Sridhara (8th cent. A.D.). This method reached Italy in the middle ages through the Arabs and was called the "*Modo per rekiego*".

7. CERTAIN MENSURATION FORMULE

The following formulae for the mensuration of a circle were stated by Umaswati (150 B.C.) in his Tattvartha Dhigama-Sutra-Bhashya[3] :

(*i*) Circumference of a circle$=\sqrt{10\,(\text{diameter of the circle})^2}$

(*ii*) Area of a circle$=\frac{1}{4}$ (circumference)$\times$(diameter)

If a denotes the arc of a segment of a circle less than a semi circle, c its chord, h its height or arrow, and d the diameter of the circle, then

(*iii*) $c=\sqrt{4h\,(d-h)}$

(*iv*) $h=\frac{1}{2}(d-\sqrt{d^2-c^2})$

(*v*) $a=\sqrt{6h^2+c^2}$

(*vi*) $d=\frac{1}{h}(h^2+\frac{1}{4}c^2)$

All the above formulae, except the formula (*v*) for finding the arrow, are restated in the Jambudvipa samasa of Umaswati. In this work, the formula corresponding to (*v*) is

$$h=\sqrt{\tfrac{1}{6}(a^2-c^2)},$$

which is the same as (*v*) in another form.

As stated earlier, the above mensuration formulae given in the work of Umaswati were not discovered by him. In fact most of these formulae were known in India, centuries before him. In the Surya Prajnapti[4] (500 B.C.) and other early Jaina works, are stated the length of the diameter and the circumference of certain circular bodies. These texts have used some of the above formulae for the computation of the circumference of the Jambudvipa (the earth) from its given diameter. According to the Jain cosmography,[5] the Jambudvipa is a circle of diameter 100,000 yojana and is divided into seven parts by a system of six mountain ranges running parallel, east to west, at regular intervals. The sacred books[6] of the Jainas (of about 500 B.C.) give the dimensions of the Jambudvipa as :

1. See Brahma-Sphuta-Siddhanta (B. S. S.) Chap. XII, p. 55.
   Brahmagupta calls it Bheda method, while others call it Vibhaga-gunana. Compare H. T. Colebrooke. "Algebra with arithmetic and mensuration from the Sanskrit of Brahmagupta and Bhashkara." London 1817, p. 61.
2. See Rule 9.
3. Tattvārtha-Dhigama-Sutra-Bhāṣhya with the commentary of Umaswati and notes of Siddhasena Gani (c. 56 B. C.) Part I.
   Edited by H. R. Kapadia. Bombay 1926, p. 258-260.
4. Surya Prajnapti. See Sutra 20.
5. Datta "Geometry in the Jaina Cosmography".
   Quellen Und Studien Zur Ges. D. Maths. Ab. B. Ed.-1 (1931) p, 245-254.
   Also See Tattvārtha Dhigama-Sutra-Bhāṣhya.
6. See. 1. Jambudvipa Prajnapti. Sutra 3.
   Ed. A. N. Upadhaya and Hira Lal Jain.
   Jain Sanskrit Sansksha Sangha. Solapur 1958.

circumference=316,227 yojana, 3 gavyuti, 128 dhanu, $13\frac{1}{2}$ angula and a little over; and area=790,569 41,50 yojana, 1 gavyuti, 1515 dhanu, 60 angula nearly,

where 1 yojana=4 gavyuti

1 gavyuti=2000 dhanu

1 dhanu=100 angula.

It may be observed here that in calculating the above values of the circumference and the area of the Jambudvipa from the formule (*i*) and (*ii*), there has been followed a principle of approximation to the value of a surd which may be expressed as

$$\sqrt{N}=\sqrt{a^2+\varepsilon}=a+\frac{\varepsilon}{2a}$$

The modern historians of mathematics, by mistake have attributed the credit of this approximate square-root formula to Heron of Alexendra[1] (3rd Cent. A.D.), but the credit for its first discovery should very rightly go to the Indians.

In Jaina work, we notice another kind of approximation. In a mixed number, the fractional part greater then $\frac{1}{2}$ is replaced by 1, while the fractional part less than $\frac{1}{2}$ is ignored. For practical purposes, the value of a quantity is often times given in round figures and the true value of that quantity is either a little more (*Kincidviseṣā dhika*) or a little less (*Kincidvi seṣoṇa*).

As stated earlier, according to Jain cosmography, the Jambudvipa is divided into seven parts. The Jambudvipa Prajnapti (500 B.C.) gives the linear dimensions of each of these parts.[2] The southern most segment of the Jambudvipa[3] is called the Bhāratavarsa (India). The dimentions of this segment, as stated in Jambudvipa Prajnapti, are :

the breadth *i.e.* the height of the circular segment is

$$=526\frac{6}{19}\text{ Yojana}$$

the length *i.e.* the chord of the segment is

$$=14471\frac{6}{19}\text{ Yojana and a little over.}$$

the length of the southern boundary of the segment *i.e.* the arc

$$=14528\frac{11}{19}\text{ Yojana.}$$

2. Jivabhigama Sutra. Sutra 82, 124.
3. Anuyogadwara Sutra. Sutra 146.
4. Jambudvipa Samasa of Umaswati (150 B. C.) Ch. I.
5. Ttrailokya dipika and Laghu Ksetra Samasa of Ratna Sekhara Suri. (1449 A. D.).

1. See Smith History II. p. 254.
2. W. Kirfel, Die Kosmographie der Inder. Bonn. 1920. p. 216.
3. See Jambudvipa Prajnapti. Sutra 10-12, 16. With the commentary of Santi Candra Gani. Ed. by Agamodaya Samiti of Mahasana. 1918.

A mountain called *Vaitadhya*, of the depth of 50 Yojana, runs through the middle of the Bharatvarsa parallel to its length. The northern and southern sides of the mountain are $10720\frac{12}{19}$ and $9748\frac{12}{19}$ yojana respectively. Further, the portions of the bounding arc and cut off by two parallel sides are given to be $\left(488\frac{16}{9}+\frac{1}{38}\right)$ yojana each. All these numerical calculations establish that most of the mensuration formulae as recorded by Umaswati were well known to the author of the Jambudvipa Prajnapti.

In the Uttra dhyana-sutra (300 B.C.), the description of *Isutpragbhara*, which resembles in form an open umbrella, *i.e.* the segment of a sphere, is :

"It is forty five hundred thousand yojana long, and as many broad, and it is somewhat more than three times as many in circumference. Its thickness is eight yojana, it is greatest in the middle and decreases towards the margin, till it is thinner than the wing of a fly".[1]

The Aupapatika-sutra[2] further specifies the circumference to be 14239800 yojana and it is also said that the depth decreases an angula for every yojana. This description suggests that the early Jains had a knowledge of mensuration of a spherical segment.

The relation between a, h and c *i.e.* the formula (v) is given in the GSS[3] of Mahavira (850 A.D.) and the Maha Siddhanta[4] of Aryabhata II (10th cent. A.D.). They have given an alternative formula which varies slightly only in the coefficient of h^2.

According to Mahavira,

$$a\ (\text{gross}) = \sqrt{5h^2+c^2}$$
$$a\ (\text{net}) = \sqrt{6h^2+c^2}$$

According to Aryabhata,

$$a\ (\text{gross}) = \sqrt{6h^2+c^2}$$
$$a\ (\text{net}) = \sqrt{\frac{288}{49}h^2+c^2}$$

The Greek Heron of Alexandria[5] (c. 200) has taken

$$a = \sqrt{4h^2+c^2}+\tfrac{1}{4}h$$
$$= \sqrt{4h^2+c^2}+[\sqrt{4h^2+c^2}-c]\frac{h}{c}$$

The Chinese ch'en Huo (died 11th cent. A.D.) used the formula[6]

$$a = c+2\frac{h^2}{d}$$

But the Indian value of 'a' is older and more accurate than the other two.

1. Uttara-Dhyana-Sutra. Chap. XXXVI, p. 59-60.
2. Aupapatika Sutra. Ed. by Leumann. p. 163-7.
3. Chap. VI. Sutra 43, $73\frac{1}{2}$.
4. Maha Siddhanta of Aryabhata.
   Ed. by Sudhakara Dividi. Banaras. 1910. Chap. XV, p, 90, 94, 95.
5. T. Heath. History of Greek Mathematics. Oxford 1921. Vol. II, p, 331.
6. Y. Mikami "The development of Mathematics in China and Japan".
   Leipzig 1913. p. 62.
   Hereafter referred to as Mikami's Chinese Mathematics.

The formula (*iii*) *viz.* $c=\sqrt{4h(d-h)}$ refers to the theorem on the geometrical properties of circles *viz.*,

"the square on the chord=the rectangle contained by the segments of the diameter perpendicular to the chord."

The formula (*iv*) is obtained by solving the quadratic equation $c^2=4dh-4h^2$. This clearly explains that the early Jainas knew how to solve quadratic equations.

8. JAINA VALUE OF $\pi(=\sqrt{10})$

The formula (1.), *viz.* circumference of a circle$=\sqrt{10\,(\text{diameter})^2}$, gives $\sqrt{10}$ as value of π. Surya Prajnapti[1] (500 B.C.), gives two values of π *viz.* $\pi=3$ and $\pi=\sqrt{10}$. The former value was given by the early writers and the later one was adopted through the early Jain literature. In the Uttaradhyayana-sutra[2] (300 B.C.), the circumference of the Jambudvipa is given to be little over three times its diameter. According to the Jivabhigama-sutra[3], corressponding to an increment of 100 in the diameter, the circumference increases by 316. This gives $\pi=3.16$. All the medieval Jaina works[4] from 500 B.C. till the 15th century A.D. used $\sqrt{10}$ as the value of π, althogh by that time more accurate value of π had been discovered by the Indians[5] It may be observed here that Professor Mikami's statement "that the value of $\pi=\sqrt{10}$ is found recorded in a Chinese work by Chong Heng (78-139 A.D.) before it appeared in any Indian work" is not correct[6].

9. THEORY OF NUMBERS

Jaina works refer to a very large number of names giving the positions (sthana or place) in the numeral system. Mahavira[7] (850 A.D.) has stated twenty-four notational places, while all other Indian mathematicians have given names for only eighteen places. The twenty-four notational places, according to Mahavira, are given below. Here the value of each succeeding place is taken to be ten times the value ot the immediately preceding place.

Eka (for 1), dasa(for 10), shata(10^2), sahasra (10^3), dasa sahasra (10^4), laksa (10^5), dasa laksa (10^6), koti (10^7), dasa koti (10^8), sata koti (10^9), arbuda (10^{10}), nyarbuda (10^{11}), kharva (10^{12}), maha kharva (10^{13}), padma (10^{14}), maha padma (10^{15}), ksoni (10^{16}), maha ksoni (10^{17}), sankha (10^{18}), maha sankha (10^{19}), ksiti (10^{20}), maha ksiti (10^{21}), ksobha (10^{22}), and finally maha ksobha (for 10^{23}).

Thus in the Jain literature, the terminology above the fourth denomination have been coined by a system of grouping and regrouping. We may note here the deviation from the vedic terminology.[8] In vedas

1. Surya Prajnapti. Sutra 20.
2. Uttara-dhayana-sutra. Chap. XXXVI, p, 59.
   Compare also Jambudvipa Prajnapti. Sutra 19.
   Trigunam Savisesam (a little over three times).
3. Jivabhigama-sutra. Sutra 112.
4. Jivabhigama-Sutra. Sutra 82, 109, 112, etc.
   Jambudvipa Prajnapti. Sutra 3.
   Bhagwati-sutra. Sutra 91,
   Tattavārtha Dhigama-Sutra-Bhāshya.
5. See Laghu Ksetra Samasa Prakarma of Ratna Sekhasa Suri (1440 A. D.) included in the Prakarma Ratnakara.
   Ed. by Bhimaseiha Maraka. Bombay 1881. Verse 187.
6. Mikami's Chinese Mathematics. p, 70.
7. See G. S. S. Chap. I. p, 63-68.
8. See Yajurveda Samhita. Chap. XVII. 2.

(about 3000 B.C. or probably much earlier), distinct and special names for each of the units of different denominations have been taken, *viz.* eka (for 1), dasa (10), shata (10^2), sahasra (10^3), ayuta (10^4) niyuta (10^5), prayuta (10^6), arbuda (10^7), nyarbuda (10^8), somudra (10^9), madhya (10^{10}), anta (10^{11}), and parardha (10^{12}).

The combination terms used by the Jainas indicate that sufficiently large numbers were of frequent usage and that is the reason why the combination terms were preferred over the distinct terms as given in the vedas.

The Jainas and the Buddhists employed fantastically large numbers in the measurement of space and time. No nation has used such large numbers. By the conception of 'Shirsha Prahelika' the Swetamber[1] Jains suggested a number of the order of $(8400,000)^{28}$ for a certain measurement of time. Bhaskara Hema Chandra[2] (b. 11th Cent. A.D,), the commentator of Anuyoga dwara-sūtra (about 100 B.C.), has stated that this number *viz.* $(8400,000)^{28}$ or $(84^{28} \times 10^{140})$ occupies 194 notational values. The Jainas used 1 Samaya as the smallest unit of time.

The following table[3], according to the Swetamber Jains*, exhibits the complete series of 36 other units of time between one Samaya and one Shirsha Prahelika, the smallest and the greatest units respectively.

TABLE OF THE UNITS OF TIME (BY SWETAMBARA JAINS)

An infinite number of Samayas	= 1 aylika
$4446\frac{2458}{3773}$ aylika	= 1 pran
7 prans	= 1 stoka
7 stoka	= 1 lava
$38\frac{1}{2}$ lava	= 1 ghari
2 ghari	= 1 muhurta (=48 minutes)
30 muhurta	= 1 ahoratra
30 ahoratra	= 1 masa (month)
12 masa	= 1 varsh (year)
8400,000 varsh	= 1 poorvang
,, poorvang	= 1 poorva
,, poorva	= 1 trutitang
,, trutitang	= 1 trutit
,, trutit	= 1 addaang
,, addaang	= 1 ad
,, ad	= 1 avvang

35. See Anuyoga-dwara-sutra. Chap. on Samaya.
36. See Anuyoga-dwara-sutra. Sutra 116.
37. See Bhagwati Sutra. Sutra 6, 7. p, 246-7.

*See Vishwa Prahilika by Muni Shri Mahandra Kumar Ji Jain. Jan. 1969.

,,	avvang	=	1 avava
,,	avava	=	1 hoohookang
,,	hoohookang	=	1 huhuk
,,	huhuk	=	1 utplang
,,	utplang	=	1 uptal
,,	utpal	=	1 padmang
,,	padmang	=	1 padma
,,	padma	=	1 nalinang
,,	nalinang	=	1 nalin
,,	nalin	=	1 arth nipurang
,,	arth nipurang	=	1 arth nipur
,,	arth nipur	=	1 ayutang
,,	ayutang	=	1 ayuta
,,	ayuta	=	1 prayutang
,,	prayutang	=	1 prayuta
,,	prayuta	=	1 nyutang
,,	nyutang	=	1 nyuta
,,	nyuta	=	1 chulikang
,,	chulikang	=	1 chulika
,,	chulika	=	1 shirsha prahelikang
,,	shirsha prahelikang	=	1 shirsha prahelika

According to the Digambar Jains, there is a change in the units of time after the 'Varsh'. However, the complete table of the units of time as given by the Digambar Jains is given below. It may be observed here that according to the Digambar Jains, the greatest unit of time is 'achlatma' and its value is $84^{31} \times 10^{90}$, and that there are 39 other units in between the smallest and the largest units of time.

TABLE OF THE UNITS OF TIME (BY DIGAMBAR JAINS)

An infinite number of samayas = 1 aylika

$4446\frac{2458}{3773}$ aylika	=	1 pran
7 pran	=	1 stoka
7 stoka	=	1 lava
$38\frac{1}{2}$ lava	=	1 ghari
2 ghari	=	1 muhurat (= 48 minutes)
30 muhurat	=	1 ahoratra

30 ahoratra	=1 maas
12 maas	=1 varsh
8400,000 varsh	=1 poorvang
8400,000 poorvang	=1 poorva
84 poorva	=1 parvang*
8400,000 parvang	=1 parva*
84 parva	=1 nyutang
8400,000 nyutang	=1 nyuta
84 nyuta	=1 kumudang
8400.000 kumudang	=kumud
84 kumud	=1 padmang
8400,000 padmang	=1 padma
84 padma	=1 nalinang
8400,000 nalinang	=1 nalin
84 nalin	=1 kamlang
8400,000 kamlang	=1 kamal
84 kamal	=1 trutitang
8400,000 trutitang	=1 trutit
84 trutit	=1 attang
8400,000 attang	=1 attat
84 attat	=1 ammong
8400,000 ammong	=1 ammom
84 ammom	=1 hahang
8400,000 hahang	=1 haha
84 haha	=1 huhang
8400,000 huhang	=1 huhu
84 huhu	=1 latang
8400,000 latang	=1 lata
84 lata	=1 mahalatang

*It seems that those two terms were left out in Triloya Pannati.

See :	Triloya Pannati	4-293, 307
	Adipuran	3-218, 227
	Lok Vibhag	5-139, 148

8400,000 maha latang	=	1 maha lata
,, maha lata	=	1 shri kalpa
,, shri kalpa	=	1 haste prahelit
,, haste prahelit	=	1 achlatma

9.1. CLASSIFICATION OF NUMBERS

The introduction of such large numbers led the Jainas to the conception of infinity. The Jainas, like the Greece,[1] do not consider 'unity' a number (*Eko* gaṇanāsamkhyā *na upeti*). In Anuyogadwara-sutra[2] (about 100 B.C.), the whole set of numbers is divided into three groups :

1. Sankhyeya ("numerable")
2. Asankhyeya ("in-numerable")
3. Ananta ("infinite")

and the highest numberable number is defined as :

"Consider a trough of the size of Jambudvipa, whose diameter is 100,000 Yojana and the circumference is 316227 Yojana, 3 gavyuti, 128 dhanu, 13½ angula and a little over. Now fill up this trough with white mustard seeds counting them one after another. In the same manner fill up with mustard seeds other troughs of the sizes of the various lands and seas of the Jain Cosmography and count the seeds one after another. The total number of mustard seeds will still be less than the highest numerable number. Thus it is difficult to reach the highest number amongst the numerables. The highest numberable number of the early Jainas corresponds to what is called Aleph Zero or Aleph-Null in modern mathematics."

Let *N* be the highest numerable number as defined above. For numbers beyond that, the Anuyogadwara-Sutra suggests the following sequence of operations :

$$
\begin{aligned}
&2, 3, \ldots\ldots\ldots\ldots\ldots\ldots N,\\
&(N+1), (N+2), \ldots\ldots\ldots\ldots [(N+1)^2-1],\\
&(N+1)^2, (N+2)^2, \ldots\ldots\ldots\ldots [(N+1)^4-1],\\
&(N+1)^4, (N+2)^4, \ldots\ldots\ldots\ldots [(N+1)^8-1],\\
&(N+1)^8, (N+2)^8, \ldots\ldots\ldots\ldots [(N+1)^{16}-1],\\
&(N+1)^{16}, (N+2)^{16}, \ldots\ldots\ldots\ldots [(N+1)^{32}-1],\\
&(N+1)^{32}, \ldots\ldots\ldots\ldots
\end{aligned}
$$

It may be observed here, that in the above classification of numbers there is an attempt to defin numbers beyond Aleph-Zero. The theory of such numbers was fully developed by George Canter in 1883. The fact that an attempt was made in India by the Jaina mathematicians to define such numbers in the 1st century of the Christian era is really commendable.

The Sthananaga-Sutra[3] before (300 B.C.) gives the following interesting classification of infinity (ananta) :

"Know that infinity is of five kinds, such as infinite in one direction, infinite in two directions, infinite in superficial expanse, infinite in all expanse, infinite in eternity."

1. See, Smith's History of Mathematics. Vol. II. P, 26.
2. See Sutra 146.
3. ,, ,, 462.

This shows that the Jainas combined the idea of infinity with that of division, defining infinity in one, two or three and infinite directions.

10. LAWS OF INDICES

In Anuyogadwara-Sutra[1], one finds certain interesting terms for higher powers, integral as well as fractional, particularly the successive squares (Varga) and square-roots (Varga mula). According to this sutra, for a quantity 'a'

the prathma-varga (first square) of 'a' means a^2,

the dvitiya-varga (second square) of 'a' means $(a^2)^2$ *i.e.* a^4.

the tritya-varga (third square) of 'a' means $(a^4)^2$ *i.e.* a^8.

In general, the nth varga of 'a' means $a^{2\times2\times2\times\cdots\cdots\text{n times}}$ *i.e.* a^{2^n}

Again, the prathma-varga-mula (first square-root) of 'a' means $\sqrt{a}$ *i.e.* $a^{\frac{1}{2}}$,

the dvitiya-varga-mula (second square root) of 'a' means $\sqrt{\sqrt{a}}$ *i.e.* $a^{\frac{1}{4}}$,

the tritya-varga-mula (third square root) of 'a' means $\sqrt{\sqrt{\sqrt{a}}}$ *i. e.* $a.^{\frac{1}{8}}$

In general, the nth varga-mula of 'a' means $a^{\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\cdots\cdots\text{n times}}$ *i.e.* $a^{\frac{1}{2^n}}$

The Anuyogadwara-sutra (about 100 B.C.) gives only in positive or negative powers of 2. But the Uttaradhyana-sutra[2] (300 B.C.) gives the other powers. In the later sutra is used the multiplicative instead of the additive principle. Thus the second power is called varga ("square"), the third power as ghana ("cube"), the fourth power as varga-varga ("square-square"), the sixth power as ghana-varga ("cube-square"), and the twelfth power is called ghana-varga-varga ("cube-square-square").

In Anuyogadwara-sutra[3], we come across with statements such as :

1. "the first square-root multiplied by the second square-root, or the cube of the second square-root." Expressed in symbols, this means

$$a^{\frac{1}{2}} \times a^{\frac{1}{4}} = (a^{\frac{1}{4}})^3.$$

2. "the second square-root multiplied by the third square-root, or the cube of the third square-root". Expressed symbolically, this means

$$a^{\frac{1}{4}} \times a^{\frac{1}{8}} = (a^{\frac{1}{8}})^3.$$

According to Anuyogadwara-sutra[4], the total population of the world is a number which in terms of the denominations koti-koti etc, occupies twenty-nine places (sthana). It is a number which will be obtained on multiplying the sixth square (of two) by the fifth square, or a number which can be divided (by two) ninety-six times.

1. See Sutra 142.
2. ,, Chap. XXX 10, 11
3. ,, Sutra 142.
4. , ,, 142.

Thus the total population of the world is $2^{64} \times 2^{32} = 2^{96}$
$=79, 228, 162, 514, 264, 337, 593, 543, 950, 336.$

This figure has twenty-nine digits and is divisible (by two) ninety-six times.

All the above is conclusive to establish that the early Jainas knew the law of indices *viz* :

$$a^m \times a^n = a^{m+n}$$
$$(a^m)^n = a^{mn},$$

where *m*, *n* may be integral or fractional.

11. Permutations and Combinations

It is a very important topic in mathematics. Its earlist use, as one of the several topics for discussion in mathematics, is traceable only from the time of the Jaina canon Sthananga-sutra[1] (before 300 B.C.), The general formulae were given later by Mahavira (850 A.D.) in his GSS. In fact Mahavira[2] is the world's first mathematician to give the general formulae

$${}^nC_r = \frac{n(n-1)\,(n-2)\,\ldots\ldots\ldots\ldots\ldots(n-r+1)}{1.2.3\ldots\ldots\ldots\ldots\ldots\ldots\ldots r} = \frac{n!}{r!\,(n-r)!}$$

$${}^rP_r = n\,(n-1)\,(n-2)\ldots\ldots\ldots\ldots\ldots\ldots\ldots(n-r+1).$$

for the total number of combinations of *n* things taken *r* at a time and for the total number of permutations of the *n* things taken *r* at a time respectively.

In Anuyogadwara-sutra[3] (about 100 B.C.), the number of permutations of six things is given by $1 \times 2 \times 3 \times 4 \times 5 \times 6$. Silanka[4] (9th cent. A.D.), the Jain commentator of the Anuyogadwara-sūtra, has reproduced, from some mathematical texts, the rules for the permutations and combinations. The rule for determining the total number of transpositions that can be made with a specific number of things (bhede-samkhya-parijnanaya) is: "Beginning with unity upto the number of terms, multiply successively the (natural) numbers : That should be known as the result in the calculation of permutations and combinations (vikalpa-gaṇita)".

Thus the total number of permutations that can be made from *r* given different thing taken all at a time is

$$rp_r = 1.\,2,\,3.\,4.\,5.\,\ldots\ldots\ldots\ldots\ldots\ldots\,(r-1) = r!.$$

The other rules for finding the actual spread or representation (*prastāranayanopāya*) are : "The total number of permutations being divided by the last term, the quotient should be divided by the rest : They should be placed successively by the side of the initial term in the calculation of permutations and combinations."

Simple problems are stated in the Bhagwati-sūtra[5] (300 B.C.). The corresponding Indian expressions used for the modern terms 'taken one at a time', 'taken two at a time', 'taken three at a time' etc. are respec-

1. See Rule 747.
2. „ GSS Chap. VI Rule 218, p, 94.
3. „ Rule 103, 115, 116 and others.
4. „ Vide his Commentary on the Sutra.
   Krtanga-sutra, Samaya dhyayana.
   Anuyogadwara. Vers. 28.
5. See Sutra 314.

tively Eka samyoga, dvika samyoga, trika samyoga etc. Although some methods of finding out the permutations and combinations of certain things were known by the time the Bhagwati-sutra was written, yet the definite formulation of any mathematical rule is traceable only from the time of the Anuyogadwara-sutra (about 100 B.C.).

In Sushrutas[1] medicinal work (about 600 B.C.), it is stated that out of six different rasas (*viz.* sweet, bitter, sour, saltish, hot, astrigent) 63 combinations can be obtained by taking the rasas one at a time, two at a time, three at a time etc. This gives the respective number of combinations 6, 15, 20, 15, 6 and 1, which obviously sum upto 63.

There are similar calculations of the groups that can be found out of the different instrument of senses (karanas), or of the selections that can be made out of a number of males, females and eunchs, of the permutations and combinations in various other things.[2] In all the cases the results are given as could be obtained with the help of the above general formulae given by Mahavira (850 A. D.).

Thus the word vikalpa for combinations is traceable before the advent of Jainism. Although the notion of permutations and combinations is traceable in India even prior to Jainism, yet the credit goes to the early Jainas for the simple two reasons—firstly for treating the subject as a separate topic in mathematics and secondly for working out the general formulae by the time of Mahavira (850 A. D.).

12. CONCLUDING REMARKS

The original mathematical works of the Jains have not come to light and a considerable amount of search and research about the Jaina manuscripts is, therefore necessary. In fact, there are three main difficulties in the study of ancient Indian mathematics *viz.*

1. the difficulty of getting original works, some of which are not available in India,
2. the difficulty of the language—ancient mathematical works are in Sanskrit, and most of them are in poetry and not in prose, which makes it all the more difficult to understand them and lastly
3. the writers of original scientific treatises are generally very brief.

Their aim was to just indicate the general outline of procedure and to leave the details to be worked out by the interested worker in the field. Some writers have given bare rules without demonstrations or examples, and the whole thing is so condensed that it is often difficult to interpret their meaning by one who is not a mathematician and a Sanskritist at the same time.

1. See Sushruta Samihita, Chap. LXIII.
   Rasabheda Vikalpadhaya.
2. See Jambudvipa Prajnapti. XX, Sutras 4, 5.
   Anuyogadwara-Sutra. Sutras 76, 96, 126.

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