Ganitasar-Sangraha of Mahaviracharya

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Aryabhata the elder (c. 510 A. D.), Brahmagupta (c. 628 A. D.), Mahaviracharya (c. 850 A. D.) and Bhaskaracharya (c. 1150 A. D.) are the most eminent mathematicians of ancient India.

Mahaviracharya, the author of the Ganitasara Samgraha, lived in a period well-known, in the history of South India, for its prosperity, political stability and academic fertility. He was a contemporary and enjoyed the patronge of Nripatunga or Amoghavarsha (815–877 A. D.) of the Rashtrakuta dynasty. Nripatunga was ruling at Manyakheta but his kingdom extended far northwards. His capital was a centre of learning. He was not only a mightly ruler, but also a patron of poets and himself a man of literary aptitude and attainments. A Kannada work, Kavirajamaraga, on poetics is attributed to him. He was a great devotee of Jinasena (the author of Adipurana and Parsvabhyudaya) whose ascetic practices and literary gifts must have captivated his mind. He soon became a pious Jaina and renounced the kingdom in preference to religious life as mentioned by him in his Sanskrit work, the Prasnottara–ratnamala and as graphically described by his contemporayy Mahaviracharya in his Ganitasara Samgraha.

Mahaviracharya combines the discipline of seasoned mathematician with the warm and vivid imagination of a creative poet. He skilfully summarizes all the known mathematics of his time into a perfect text–book which was used for centuries in the whole of Southern India. He states rules clearly and precisely. He simplifies and sharpens many processes. He generalises many a theorem shedding light on new aspects by apt illustrations. Ganitasara–Samgraha is a veritable treasury of problems many of which are characterised by mathematical subtlety, poetic beauty and delicate hint of refined humour, qualities so rare in a mathematical text book. It is difficult to decide, in a textbook what is old and what is the original contribution of the author.

Here is a brief survey of the contents of the book :

Chapter I opens with the salutation to Lord Mahavira, the twentyfourth Tirthankara of the Jainas, who by his knowledge of the science of the numbers illuminates the three worlds. This is followed by a warm and handsome tribute of gratitude paid to his royal patron Amoghavarsha. After this, comes the most enthusiastic and unique panegyric ever bestowed on the science of Mathematics. Then we have measures used, names of operations and numerals. Rules governing the use of negative numbers are correctly stated: those regarding the use of zero may be stated in modern notation thus :

$$a \pm o = a; \qquad a \times o = o; \qquad a \div o = a.$$

The last part is obviously wrong. As regards the square root of a negative number, the author observes that since squares of positive and negative numbers are positive, square root of a negative number cannot exist. Considering the limitations of his time, Mahaviracharya could not have reached a more sensible conclusion. We may note, in this context, the necessary extension of the concept of number which assimilates square roots of negative numbers into the number system, was achieved as late as in 1797 by C. Wessel a Norwegian Surveyor (Bell's "The Development of Mathematics" Page 177).

Chapter II deals, in respect of integers, with operations of multiplication, division, squaring and its inverse, cubing and its inverse, arithmetic and geometric series.

Problem II. 17. In this problem, put down in order (from the unit's place upwards) 1, 1, 0, 1, 1, 0, 1 and 1, which (figures so placed) give the measure of a number and (then) if this number is multiplied by 91, there results that necklace which is worthy of a prince. The 'Necklace' referred to, may be displayed thus :

$11011011 \times 91 = 1002002001.$

Two more 'garlands worthy of a prince' are : (II. 11, 15):

$333333666667 \times 33 = 11000011000011;$

and $752207 \times 73 = 11,111,111.$

Chapters III and IV are devoted to elementary operations with fractions. Mahaviracharya has paid considerable attension to the problem of expression of a unit fraction as the sum of unit fraction. This problem has interested mathematicians from remote antiquity (Ahmes Papyrus, 1650 B. C.). Here are three relevant problems (II 75, 77, 78) set in modern notation.

$$(1) \quad 1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{3^2} + \cdots\cdots + \frac{1}{3^{n-2}} + \frac{1}{3^{n-2}.3}$$

$$(2) \quad 1 = \frac{1}{2.3.\frac{1}{2}} + \frac{1}{3.4.\frac{1}{2}} + \cdots\cdots + \frac{1}{(2n-1)2n.\frac{1}{2}} + \frac{1}{2n.\frac{1}{2}}$$

$$(3) \quad \frac{1}{n} = \frac{a}{n\,(n+a_1)} + \frac{a_2}{(n+a_1)\,(n+a_1+a_2)} + \cdots\cdots$$

$$+ \frac{a_{r-1}}{(n+a^1+a_2+\ldots\ldots+a_{r-2})\,(n+a_1+a_2+a_{r-1})}$$

$$+ \frac{a_r}{a_r\,(n+a_1+a_2+\ldots\ldots a_r)}$$

Problem IV. 4: One third or a herd of elephants and three times the square root of the remaining part (of the herd) were seen on the mountain slope; and in a lake was seen a male elephant along with three female elephants. How many were the elephants there ?

Here is a sample of monkish humour:

Chapter V treats 'Rule of three' and its generalised forms.

Chapter VI. Having created the arithmetical apparatus in the earlier chapters in this long chapter, Mahaviracharya applies it to solving many problems which one encounters in life such as money lending, number of combinations of given things, indeterminate equations of first degree etc.

Problem (VI. 128) : In relation to twelve (numerically equal) heaps of pomgranates which having been put together and combined with five of those (same fruits) were distributed equally among 19 travellers. Give out the numerical measure of (any) one heap.

Problem (VI. 218): The number of combinations of n different things taken r at a time is.

$$\frac{n(n-1)(n-2)\cdots\cdots(n-r+1)}{1.2.3.\ldots\ldots\ldots r} \text{ or } - \frac{n!}{r!\,(n-r)!}$$

It is interesting to note that this general formula was discovered in Europe as late as in 1634 by Herigone (Smith's History of Mathematics, Vol II). We may also recall here that the number 7 which occurs in Saptabhangi provides a simple example in the theory of permutations and combinations. A layman can verify that he can form seven and onlv seven different combinations of three distinct objects. Jainas have been using mathematics freely in their sacred literature from very remote antiquity. The above example supports this fact.

Problem (VI. 220) : O friend, tell me quickly how many varieties there may be, owing to variation in combination of a single-string necklace made up of diamonds, supphires, emeralds, corals and pearls ?

Problem (VI. 287) : What is that quantity which when divided by 7, (then) multiplied by 3, (then) squared, (then) increased by 5, (then) divided by 3/5, (then) halved and then reduced to its square root, happens to be 59.

Note the sheer devilry of it :

In chapters VII and VIII problems on measuration are treated. Some of the formulas used are noted here :

(1) The Pythagorean formula for the sides of a right-angled triangle is $a^2 = b^2 + c^2$ where a is the hypotenuse.

(2) Area of $\triangle$ ABC is

$\sqrt{s(s-a)(s-b)(s-c)}$ where $2s = a + b + c$.

(3) The area and the diagonals of a quadrilateral ABCD are:

$\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $2s = a + b + c + d$:

$\sqrt{\frac{(ac+bd)\quad(ab+cd)}{ad+bc}}$ $\sqrt{\frac{(ac+bd)\quad(ad+bc)}{ab+cd}}$.

It is unfortunate that both Mahaviracharya and his predecessor Brahmagupta made the common mistake of not mentioning the fact that these formulas hold for cyclic quadrilaterals only.

(4) $\pi = 3$ or $\sqrt{10}$

(5) The circumference of an ellipse whose major and minor axes are of lengths 2a and 2b is $\sqrt{24 b^2 + 16 a^2}$ which redues to $2\pi a \sqrt{1 - \frac{3}{8} e^2}$ where e is the eccentricity. It is difficult to imagine, how Mahaviracharya could attain such a close apprximation without the help of the powerful tools available to us.

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