The Jaina Mathematical Philosophy

☐ Prof. L. C. Jain

"Early Greek geometers, passing from the empirical rules of Egyptian land-surveying to the general propositions by which those rules were found to be justifiable, and then to Euclid's axioms and postulates, were engaged in mathematical philosophy......" —Bertrand Russell

1. Introduction

Bertrand Russell (1872-1970) appears to have worked on an adventurous path of introducing philosophy through mathematics, in place of logic. He maintained that mathematics ought to be recognized as symbolic logic and he contributed to this end through his world famous work "Principia Mathematica" in collaboration with Whitehead.

In the history of philosophy, there is a digression marked with the Jaina philosophy. It had an approach to a set-theoretic system theory known as Karma theory on which Ṣaṭkhṇḍāgama, Mahābandha and Kasāyapāhuḍa texts were compiled round about the beginning of the Christian era.

The Jaina school developed post-universal measures through existential sets (cardinals as simile measures) and constructive sets (ordinals as number measures), both detailed as upamā māna and saṃkhyā māna in the text of the Karṇānuyoga group, viz. the Tiloyapaṇṇattī (C. 5th century A. D.) the Trilokasāra (C. 11th century A. D.) and so on.

These works comprise the mathematico-philosophic pursuits, carried through various commentaries, of which the available texts are the Dhavalā and Jayadhavalā (C. 9th century A. D.). They, a priori, had the logical trends. But when Nemicandra Siddhānta Cakravartī (C. 11th century A. D.) abstracted them in mathematical form, known as the Gommaṭasāra, and the Labdhisāra, the succeeding commentators required a well developed symbolism for their elaborate studies.

The pioneers in this field appear to be Mādhavacandra Traividya and perhaps Cāmuṇḍarāi (11th century A. D.), who might have before them the works or their predecessors, mainly Kundakunda, (C. 3rd century A. D.) Samantabhadra, Yativṛṣabha, (C. 5th century A. D.) and Akalaṅka (C. 8th century A. D.). It was to the credit of Keśava varṇī (C. 14th century A. D.) to consolidate all these as Karṇātavṛttis[1], further translated as Jīva tattva

1. *Gommaṭasāra (vols. 1-4), Bharatiya Jñāna Pītha, New Delhi, 1981, onwards.*

pradīpikā into Sanskrit. Ṭoḍaramala (C. 18th century A. D.), basing his Samyakjñānacadrikā commentary on all their works, in the eighteenth century, bifurcated the mathematical philosophy of the commentaries into the philosophy as lay man texts and the mathematics as artha saṃdṛṣṭi (symbolic norms) in two chapters, set introductory to the Gommaṭsāra and Labdhisāra (inclusive of Kṣapaṇāsāra), of 11th century A. D.

2. Modern Context

The necessity for a mathematical philosophy in the modern era arose due to class of paradoxes and antinomies in the theory of sets founded by Georg Cantor, through his researches beginning since 1864. He was opposed by Dedekind and his school so much so that his concepts of infinities which were distinguished from philosophical infinity as proper infinities (as they could be compared mathematically as smaller or greater) received foundational blow. Georg Cantor was so puzzled that ultimately he died in an asylum for the lunatics.

Burali-forti (Italian) and Russell round about the beginning of the 20th century, showed the paradoxes of the set theory of Cantor. Attempts then began with Russell to build up the set-theories in order to avoid the paradoxes and Contradictions through intuitionistic, logistic and Hilbert's formalistic approaches.

The Jaina school of thought had also a set theory[1] in which extensive use of mathematically comparable infinities were used through monads or units introduced for space, time, phases, matter, motion. These were denominated as point (pradeśa), instant (samaya), indivisible corresponding sections (avibhāgī praticcheda) of various types for Yogic and Kāṣāyic actions as well as certain phases of a bios (pariṇāmas).

It was natural to avoid contradictions and paradoxes of infinities in giving mathematical operations among them.[2] Logic, therefore, took its role towards consistent inferences, for which syādvāda was a process of eliminating the inconsistencies and contradictions. The omniscience was regarded a supreme but adaptable set of indivisible-corresponding-sections of all knowledge of all fluents (dravyas) supposed to exist through their own controls (guṇas) and own events (paryāyas) from ab-aeterno to ad-aeterno.

This knowledge was applied to their Karma[3] system theory, which was the main source leading to a mathematical-philosophy in a naive form.

1. *Jain, L. C., Set Theory in Jaina School of Mathematics, I. J. H. S., Vol. 8, no. 1, 1973, 1-27.*
2. *Jain, L. C., On Certain Mathematical Topics of the Dhavalā Texts; I. J. H. S, vol. 11; no. 2, 1976, pp. 85-111.*
3. *Jain, L. C.. System Theory in Jaina School of Mathematics, I. J. H. S., vol. 14, no. 1, 1979, pp. 29-63.*

3. Set Theoretic Approach

Gödet, a great set theorist, concluded that it is impossible to establish a meta-mathematical proof of the consistencies of a system comprehensive enough to contain whole of arithmetic. He found that Principia or any other system within which artihmetic can be developed is essentially incomplete. According to him Principia Mathematica must contain "undecidable" formulas, as much also contain axiom systems for *set theory* when formalized by addition of axioms and rules of conclusion of the calculus of logic.

In spite of the incompleteness of a system, it does work for all branches of studies, for example the set theory so developed has been applied to science and other fields where abstract methodology has been evolved. Similary, strange enough to find, the naive set theory is found to have been applied to Jaina religious philosophy of Karma (action).

The fundamental word for set in Jaina texts is rāśi, with its synonyms, samūha, ogha, puñja, vṛnda, sampāta, Samudaya, piṇḍa, avaśeṣa, abhinna and Sāmānya, as found in the Dhavalā texts. The sets have been classified as : unitary elements of sets, fundamental measure units of sets, fixed fluent sets, point sets, instant sets, smallest, biggest and intermediary sets, null sets, indivisible-corresponding sections sets of controls and events, transfinite sets, Karmic sets : varga, vargaṇā, spardhaka, guṇa hāni (viz. varieties, varieform, supervarieform, geometric regression) and variable sets under the methods of treatment of sets, there are eight analytical methods, the method of reductio-ad- absurdum, the method of one-one correspondence for comparing transfinite sets. The analytical methods consist of the methods of measure (pramāṇa), reason (karaṇa), explanation (nirukti), abstraction (vikalpa), cut (Khaṇḍita), division (bhājita), spread (viralana) and removal (apahṛta). There is also the approach through the laws of indices, logarithms fraction and square piling.

Comparability in syntopology is still under active investigation for existential and constructive sets and the Jaina works of karma theory have various comparability results for various types of sets, known as alpabahutva, at regular steps, whereever needed for elaboration of ideas and concepts of cardinality and ordinality of sets under description. Alpabahutva has been called the very nature of numbers ends' of three types : about bios, about non-bios, and about mixed. They are detailed into own's place, in other place and in general. The relations used in comparability of quantities are as follows: small, equal, smallest, non-existent, distinctly great, distinctly small, summable times, non-summable times, infinite times, numerate or innumerate part, decrease and increase, least mild, and most intense etc.[1]

1. *Jain, L. C., Perspective of the System Theoretic Technique in Jaina School of Mathematics between 1400-1800 A.D., Jaina Journal, Calcutta, vol. 13, no. 2, 1978, pp. 49-66.*

Out of the fourteen topological sequences (Dhārās), the three dyadic sequences, standing with dyadic forms and reaching the set of omniscience are very important. All of them make use of the well-ordering theorem.[1] Sequential relations in each other are found through comparison and logarithms.

The various operational treatment of the sequences etc., appear to lead us to certain antinomies, paradoxes and fallacies. Yet the Zeno's paradoxes can be easily explained away by the Jaina mathematical principles of postulating finite points in finite segments although in a finite segment in analytical methods, infinite set could also be established as a representation.[2] Some of the paradoxes could be explained away through comparability and the sequences. The set of instants in the future time is stated to be infinite times that of set of instants in the past time, appears to be paradoxial, Yet it has been postulated as existent. The axiomatic method is predominant.[3]

4. Systems Theoretic Approach

The Jaina Karma theory can be compared with mathematical system theory of the modern age in so far as the karma theory deals with input, output and state transition. Yet the Jaina Karma theory had evolved as a self-reproductive system. The modern science has yet to develop abstract models for self-reproductive systems.

The Karma group of texts may also be classified either as under dravyānuyoga or karaṇānuyoga group. There are Karma structural sets, universes and operators, operands as well as transforms. The Karma theory rests on the following fundamental ground :

1. The Yoga (volition) and Moha (charm) operators, with quantitative norms and measures;
2. The tetrad of measures and norms of configuration (prakṛti, points (pradeśas) or particle-numbers, life-time (sthiti) and energy levels (anubhāga-aṅśa) corresponding to various karma structures and particularly in nisusus (niṣekas) represented in Karma life-time matrices;
3. The causality concept of simultaneous events corresponding to an absolute scale of behavioral time, corresponding to bios and matter;

---

1. *Jain, L.C., Divergent Sequences Locating Transfinite sets in Trilokasara. I. J. H. S., vol. 12, no. 1, (1977), pp. 57-75.*
2. *Jain, L. C. (editor), Gaṇita Sāra Saṁgraha of Mahāvirācārya (1963), Sholapur (introduction).*
3. *Comparison of sixteen sets in Dhavalā, book 3, is worthy of attention.*

4. Yoga operator being responsible for configuration and particle bonds whereas Moha operator being responsible for life-time and energy-level bonds;
5. The dual system of the bios phase-rise and its karmic-matter phase-rise working as feed-back to prolong the working of karmic system in ordinary course;
6. Before rise of karmic display, there is a proportionate time-lag, axcept for the age configuration;
7. There is state-existence (sattva) of karmic totality of the past, transiting every present instant, depending on the action of input of yoga, charm and self effective independent phases (pariṇāmas) of the bios, time also acting as an operator for gradual decay of the nisusus;
8. There are output values and output functions, working every instant;
9. Impedance (saṁvara) also works as an input function;
10. The fluent measure, quarter (kṣetra) measure, time measure and phase measure of various sets of bios, non-bios, their merits and demerits, influx. impedance, decay, bond and emergence of all the types of karmic configurations forming statistical survey of the karmic universes;
11. The ten operational phases of bond being those of bonding, state transition, rise, premature-rise, uptraction (utkarṣaṇa), down-traction (apakarṣaṇa), transmutation (saṅkramaṇa), subsidence (upaśama), nidhatti and nikācita operations;
12. There is a sequence in which the ending of the tetrad of bond (configuration, point, life-time and recoil-energy) occurs, through reduction in Yoga and Moha as well as effective independent pariṇāmas of a bios;
13. There is a sequence of annibilation of state, and a rule of life-time cut for life-time state; a law for the down-tract and a law for reduction of recoil energy;
14. The three operators : the low tended (adhaḥpravṛtta), the unprecedent (apūrva) and the invariant (anivṛtti) being responsible for attainment of correct vision, similar to that in omniscient;
15. The complete emergence resulting in omniscience and other infinite controls.

Thus the karmic philosophy in the Jaina School emerged out as a mathematical philosophy in which the astronomical and cosmological system were also evolved under the following basis :[1]

---

1. *Jaina, L. C,, On Certain Physical Theories in Jaina Astronomy, The Jaina Antiquary, vol. 29, nos. 1/2, 1976, pp. 9-16.*

1. Divisibility ad infinitum of space and time, in physical nature is impossible;
2. An ultimate particle of matter, by virtue of motion could be existent on more than one space-point within an indivisible instant of time;
3. In nature, the physical and bios phenomena have frequency of occurence;
4. A closed path when topologically deformed, does not lose its invariant properties;
5. Seasons changing with precession of quinoxes.

The above marks the system as a principle theory which adopts the analytical method, its basic elements not being constructed hypothetically, but discovered empirically. The basic concepts and principles from the general characteristic of the natural process. Such a theory has the advantage of being logically perfect, and have a secured foundation. Yet the principles require to be powerfully supported by experience and should be logically reconcilable.[1]

5. The Mathematio-Logical Development

The Syādvāda system of predication went very deep into the evolution of expressing the karmic system theory before the development of symbolic norms (artha saṁdṛṣṭi), from the sentential and syncopted expressions for equations in qualitative and quantitative forms.

According to Yativṛṣabha[2], the suborder of third Prābhṛta of tenth Vastu, in the fifth Pūrva called jñānapravāda, is of five types : ānupūrvi, nāma, pramāṇa, vaktavyatā (assertoriality) and artha-adhikāra. Vaktavyatā suborder is of three types : sva-samaya, para-samaya and tadubhaya. The variational aspect of assertoriality transforms it into propositionality, leading logic to symbolic form.

Vīrasena (c. 9th century A. D.) quotes a verse[3], Relative to controls and events, "That fluent is one without leaving its various-own-forms and positively it is many relative to its own controls and events, without leaving its one-ness. Thus, O, Jina, the object in infinite forms is stated in sentences, in squence, through partial acceptance phase (bhāva)." He further explains. "Relative to dravyārthika naya, there is one-ness in one and many. Relative to paryayārthika naya (purport), from an arbitrary 'one' number, the remaining 'one' numbers are different, therefore there is many-ness in them.

1. *Enistain, A., Ideas and Opinions, London, 1956, pp. 227-232.*
2. *Kasāya Pāhuḍa, Calcutta, 1955.*
3. *Dhavalā, book 3, 1940.*

Relative to naigama purport, the duality etc., phases comes into being, which leads to acceptance of number—division."[1]

The mathematical import of the following logic for deciding the fineness between space and time is important. Vīrasena mentions, "Many preceptors state that the fine is that which is accumulation of many points. It has also been stated that time-measure is fine and quarter-measure is finer, because in an innumerable part of a finger, there are innumerable Kalpas. But this assertion it not eventuated, because, at such a recognition, fluent description will follow the quarter description. Doubt: How is this ? Explanation : Because in a fluent finger, compared of infinite point-like ultimate particles, relative to embedding, there is only one quarter-finger, but relative to counting, there are infinite quarter-fingers: Hence quarter is five and fluent is finer, because there are infinite quarter-fingers in a fluent finger."[2]

Mahalanobis found in Syādvāda a close relevance to the concepts of probability.[3]

According to him, the fourth syādvāda category, being a synthesis of three basic modes, the fourth denoting inexpressibility, indefiniteness or indeterminateness, supplies the logical foundations of modern concept of probability. Yet causality being a relation of determination, it is difficult how causality could not be accomodated in its usual form in Jaina philosophy, as such the assertion of Mahalanobis is subject to reexamination, for the form of the causality held in Jaina philosophy.

Haldane puts in the quantitative aspect of the indeterminate solutions of equations under investigation of syādvāda predication. He says that solutions, of equation like $x^2 = -1$, were non-assertorial, till the discovery and use of imaginary members.[4] He finds, "Search for truth by the scientific method does not lead to complete certainty. Still less does it lead to complete uncertainty. Hence any logical system which allows conclusions intermediate between certainty and uncertainty should interest scientists".

We have still to see how behavioral (vyavahāra) and deterministic (niścaya) purports find place in the universe of knowledge sets in Jainology, for description of natural phenomena. Sahajānanda Varṇī divided the

1. *Dhavalā, book 3, p. 30.*
2. *Ibid, pp. 27-28.*
3. *Mahalanobis, P. C., The Foundations of Statistics, Dilectics, vol. 8, no. 2, 15 June 1954, (Saṁkhyā, Indian Journal of Statistics, vol. 18, pts I & II, pp.183-194.)*
4. *Haldane, J.B.S., The Syādvād System of Predication, Sāmkhyā, the Ind. Journal of Statistics vol. 18, pts. 1 and 2, (record 1956), pp. 195-200.*

purport system into as many as 225 purpots.[1] This work gives an exhaustive and experimental detail for a deep study into the purport-system.

Thus one finds that what lacks today is the team for exploring the mathematical aspects of the Jaina philosophy, which has found its beautiful architect in the revelation of the Karma theory.

□□

—Secretary
Mother Universe Foundation
677, Sarafa, Jabalpur—482002 (India)

1. *Samayasāra of Kundakunda with-Sapta Daśāṅgī commentary of Sahajānanda, Meerut. 1977.*