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JAINA MATHEMATICS

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INTRODUCTION TO JAINA MATHEMATICS

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Although declared as unorthodox by some, the Jaina system of philosophy is unique and has its own characteristics which it place high among the various systems of not only India, but of the whole world. One such characteristic is the doctrine of *anekantavda* or the theory of manifold-ness. In fact no history of human thoughts and ideas will be complete without this Jaina contribution. The remarkable thing is that the Jaina thoughts seem to be as lively in the current time as they were in the past.

Ours is an era of science and technology in which mathematics plays a significant role as a powerful tool. Without forming a large number of equations and solving them accurately, a rocket or satellite cannot be launched or orbited successfully and provide communication services.

The importance of mathematics was recognised in India. In the Jaina school, mathematics played an important role in a well-rounded philosophical education. Ṛsabha, the first Tīrthankara, was known to have taught arithmetic to his eldest son, Bharata, according to the tradition. Much more use of mathematics was made in Jaina philosophy and cosmology than in any other system. According to an ancient four-fold classification of the Jaina canonical literature, the *Gaṇitānuyoga* or *Karaṇānuyoga* is just one excellent examples. From the famous Jaina mathematician Mahāvīrācārya's *Gaṇitasāra Saṅgraha* (ca. 850 C.E.) we find the following description of the universal utility of mathematics:

> In all transactions which relate to worldly, Vedic, or other similar religious affairs, calculation is of most use. In the science of love, in economics, in music and in drama, in the science of cooking, in medical science, in architecture, in prosody, poetics and poetry, in logic and grammar, and in relation to all that constitutes the peculiar value of the arts, the mathematics is held in most high esteem. In relation to the movements of the sun and other planets, in eclipses, in conjunction of the planets, in problems related to direction, position, and time, in the moon's phases, indeed in all these, the use of mathematics is most accepted. The number, the diameter, and the perimeter of islands, oceans, and mountains; the dimensions of the

habitations and halls belonging to the inhabitants of the world, between the worlds, of the *jyotrloka*, of the world of gods and of hell-dwellers and other miscellaneous measurements etc., -- all these are known through mathematics. The configuration of living beings, the span of their lives, their eight attributes and the like, their journeys and dwelling together, etc., are all dependent upon mathematics. What is the use of saying much? Whatever there is in all the three worlds with living or moving and non-moving beings cannot be comprehended without mathematics.

It was J.W.L. Glaisner who, while giving his Presidential Address to the British Association for the Advancement of Science, said in 1890 that "no subject loses more than mathematics by any attempt to dissociate it from its history." This is more so in the case of India with its continuous tradition of at least 5000 years in philosophy, literature and the sciences.

Unfortunately, India has not been given due place and credit in the writings on the history of world mathematics. One reason for this lapse is ignorance, and the other is the difficulty of access to sources both primary and secondary.

During recent years, much work has been done in the field of history of the exact sciences of India (including the Jaina contributions). As a result, a number of new findings and discoveries have been made and a vast amount of material has been published. In this regard, an academic journal -- *Gaṇita Bhāratī* (ISSN 0970-0307) was launched in 1979.

The work in the field of the Jaina exact sciences has a number of forums and institutions; guidance of the Jaina monks, who are invariably great scholars themselves. However, the number of truly devoted research workers in the field of Jaina mathematics is relatively small. Some of the reasons for this situation are caused by difficulties in understanding the ancient languages and their complex terminologies, the historical as well as the scientific methodology, and other technicalities. There are hardly any facilities in India for formal training in the history of science through various sources, and therefore, scholars who are expert in reading and understanding ancient manuscripts and who are also simultaneously learned in ancient as well as modern mathematical sciences are quite meagre in number.

When the study and investigation in the history of the oriental sciences was first started, they were mostly undertaken by Orientalists. However, it is now accepted that it is more desirable that the history of mathematical sciences be investigated by the mathematicians themselves.

In order to do this, it is necessary to become experts in both linguistic and historical methodology.

A unique scholar of Jaina Mathematics in the present is Professor Laxmi Chandra Jain (b.1926). His vast knowledge of ancient Jaina sources, and long experience has made him a great authority of the Jaina exact sciences. His extensive research investigation into that part of Jaina Mathematics called *lokottara gaṇita* (post-worldly mathematics) has shed light on several hitherto unknown aspects in terms of modern mathematical language and notation. Special mention should be made of his research project the *Labdhisāra* of Nemicandra Siddhānta-Cakravartī (c. 1000 C.E.), which was successfully completed in four volumes in 1987. The text is an advanced theory of the Jaina Karma system.

According to ancient Indian cosmography -- whether Vedic, Buddhist, or Jaina -- there is a series of successive concentric rings (*valays*) of lands and seas with Mount Meru (a sort of celestial axis) standing in the centre. But while the widths of these rings are successively halved in Buddhist descriptions, they are said to be successively doubled in the Jaina cosmography as we go further away from the centre of Mount Meru or Sumeru. In this particular case, if D is the diameter of the Jambūdvīpa, then the width of the nth ring (whether sea or land) surrounding it, is given in the following formula:

$$W_n = 2^n D$$

Then

$$I_n = 2 W_n - 3 D$$

Now the *Tiloyapaṇṇattī* defines the *khaṇḍas* (say K_n) of any ring by the relation of the following:

$$K_n = (2^2_n - 1^2_n) / D^2$$

It can be easily seen that

$$\frac{K_n}{K_1} = (2^n - 1)\, 2^{n-1} = P_n \text{ for example.}$$

Vīrasenācāry (early 9th century), in his *Dhavalā*, has given not only K_n, but also the values of P_n up to $n = 7$. The interesting thing to note is that the set of numbers P_n also includes the so-called perfect numbers.

Again, since the boundaries of the Jaina cosmographic islands and seas are found to be circular, the geometry of a circle plays an important role in Jaina *laukilak gaṇita* (worldly or practical mathematics). But a proper and correct understanding of the various canonical values of various cosmographic lengths and areas requires an insight into the ancient methods of computation which were accessible and followed.

For instance, in modern mathematics we calculate the *paridhi* (circumference) of a circle of diameter d by using the formula:

$$p = \pi d$$

where the well-known constant π (now known to be a transcendental number) can be taken to as many decimal places as we need or want. But the approach of the ancient Jaina school was somewhat different. For obtaining results of these ancient calculations, we must not only know the type of approximation of π , which was used implicitly or explicitly, but also the manner in which it was used.

For example, the *Tiloyapaṇṇattī* gives the length of the circumference of the Jambū Island -- the diameter of which is a *lakh yojanas* (100,000) to a very fine unit called *avasannāsanna skandha* where one *aṅgula* is equal to 8^{12} of these fine units. This canonical value can not, however, be obtained by simply or directly employing,

$$\pi = \sqrt{10}$$

to any desired places or degree of accuracy -- although it is this very approximation of π whose equivalent was used by the Jainas in this connection. However, a peculiar use of the rule given by

$$p = \sqrt{(10\, d^2)},$$

and then applying the formula

$$\sqrt{(a^2 + x)} = a + (x/2a)$$

for extracting the square-root properly, and finally reducing the result to the desired sub-units, can lead us to a value of circumference p which tallies exactly with the ancient canonical measure found in the text, the

Tiloyapaṇṇattī. The question of *pramāṇa* versus *utsedha aṅgula* does not seem to be involved here.

Similarly, the ancient canonical values of the areas of the various regions of the Jambū Island can be obtained by a proper and intelligent use of the rule

$$A = \sqrt{10}\,(ch/4^2)$$

Where c is the chord (*jīvā*) and h is the heights of the segment (*iṣu*) of a circle. The above rule implies the formula

$$A = \pi\, c\, h/4$$

for the area of a segment of a circle. Had Mahāvīrācārya used this typical traditional formula in his treatment of an ellipse as a double segment, he would have hit upon the true-modern formula $\pi\, ab$ for the area of an ellipse (where a and b are the semi-axes).

One of the most original contributions of the Jaina School of Mathematics is the concept of getting *vargita-saṁvargita* of any quantity to various orders. It leads to formation of very quickly increasing sequences. The (first) *vargita-saṁvargita* of x is defined by x^x and is denoted by $x\rceil$

Thus $x\rceil = x^x = y$, say.

The *vargita-saṁvargita* of the first *vargita-saṁvargita* of x is called the second *vargita-saṁvargita* of x. That is, the *vargita-saṁvargita* of y will be the second *vargita-saṁvargita* of x. It is denoted by $x\rceil^2$. Therefore,

$$x\rceil^2 = y^y = (x^x)^{(x_x)} = z \text{ say.}$$

Similarly, the *vargita-saṁvargita* of y will be called the third *vargita-saṁvargita* of x. And so on. For example, with $x = 2$ we have the following

$$2\rceil = 2^2 = 4$$

$$2\rceil^2 = 4^4 = 256$$

$$2^{\lceil 3} = (256)^{256}$$

Using the above notation, the writer of these lines was able to obtain a good lower bound for the *jaghanya parīta asaṁkhyāta* (minimal limited innumerable). It can be expressed as

$$\log [k^{\lceil N}] / (\log 8)$$

where the integer next to N is given by

$$7.9656 \times 10^{135} \text{ (nearly)}$$

and k is great than

$$12^{10^{10^{45}}}$$

It should be noted that this value of k is better than that of *ga* obtained by Muni Mahendra Kumar II in his *Viśvaprahelikā* (Bombay, 1969), and of course, a part of the mathematics of the Jaina cosmos that requires a strenuous imagination. □

Mathematical Philosophy In Jaina Thought

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"he words or the languages, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be voluntarily reproduced and combined."
- Albert Einstein, Ideas and Opinions, Calcutta, 1979.

The words, "Mathematical Philosophy," seem to have originated with Bertrand Russell (1872 - 1970), author of the *Principia Mathematica.* It was co-authored with Alfred Whitehead. Russell, who. created "Russell paradox" in relation to the fringes of the set theory of the infinities as propounded by Georg Cantor (1845 -1918). According to Cantor, the early Greek geometers passing from the empirical rules of Egyptian land-surveying to the general propositions by which those rules were found to be justifiable, became engaged in -- especially after dealing with Euclid's axioms and postulates -- a new type of existential thought, mathematical philosophy.

More recently it has been observed that the early Jaina School of Mathematics was also engaged in a similar pursuit as is demonstrated in the mathematical philosophy of Karma theory with its mathematical discourse of symbolism contained in the *Purva* (ca. 2nd-5th century C.E.) texts that were in the hands of the Digambaras.

Pandita Todaramalla of Jaipur (c. 1721-61) was the last worker in this particular field. Due to his great accomplishment, we now have a guide to the non-universal mathematics of Jaina Karma theory contained in such works like the *Dhavalas* of Virasenacarya (begun in the year 792 and completed on the 8th of October 816 C.E), the *Gommatasara*, the *Trilokasara* of Nemicandra Siddhantacakravarti (ca. 955-985 C.E.), the *Tilyapannatti*, and works credited to Madhavacandra Traividya (c.12th century) and Kesava Varni (c. 13th century). The mathematical philosophy of Karma theory with its mathematical discourse of symbolism was extensively researched -- Project of Labdhisara at the Indian National Science Academy, New Delhi, 1984 through 1987.

It was already felt by Boole, Frege and Russell that deeper realms of philosophy could be approached only through words or symbols which could express the propositions between the truth and

untruth. The parallel to this is the concept of *syadvada* in the Jaina philosophy. Thus the status of an object being relative to different points of view, a single proposition about its state marred the prospects of its description in various aspects in the old philosophies.

The Jaina philosophy, however, was free from this mono-ended pursuit and it followed a poly-endedness. This led to the existential and constructive spheres of the innumerate and the infinities in a proper and simple way through its set theory (*rasi siddhanta*). The secret of the mathematical philosophy in the Jaina School thus lay in their attempt to give a new shape to the expressions in logic and intuition. This was achieved with the word "*syat*" in the course of the *parikarmastaka*, and not only among the finite sets, but, also for the innumerate and infinite sets of various comparabilities.

Today the problem of the comparability is still unsolved in the modern set theory of infinities of various types. In the Jaina set theory there are not only the constant sets but also the variable sets scaling the infinities of Jaina Karma theory through constructions and other analytical methods. The various types of units, measures and calculations between them were needed in their Karma system and cybernetics which was an aggregate of various sub-systems and groups of operations to annihilate the Karma state matrix. Therefore, the School had its own formalised symbolism and symbolic logic, that much like Russell later, became a Karma theory via mathematics.

The Innumerate in Jaina Set Theory: A Philosophical Sub-System

Cantor's theory of sets, had to face the contradictions, antinomies, and inconsistencies as any theory has to face for its survival. His sets included, not of the philosophies, but of proper characteristics that could prove that a set, though infinite, could be greater than another set, as well as that it could be constructed thorough the principle generalised induction. Comparability between infinite set began a new arena of research that went beyond the old philosophical domain in which there was no place to compare infinities (e.g., improper mathematical infinities for their smallness or greatness).

With such a new prospect of the infinities, the Jaina Karma philosophy took a new form. Through various sequences ranging from unity to the supreme sets of omniscience (*kevala jnana*), the Jainas located the terms of various types of sets involved in the calculations of annihilation of the perpetual karmic cycle of births and deaths. They filled the gaps between such sets as those which had the number of members as numerate, innumerate, and infinite.

The Jaina School of mathematical philosophy, therefore, took a positivistic approach to introducing the innumerate and the infinite. They were meant to explain the endless processes from *ab aeterno* to *ad infinitum*, the relations between various sets involved in the realities of various life types. They had to find, mathematically, a path to perpetual immortality in which there was neither births, rebirths, or the agonies They were looking for a way to achieve the perpetual bliss, infinite power and total knowledge.

To do this, they created the indivisible system of units, as the indivisible instant (*samaya*) and the indivisible space (*pradesa*), this is somewhat similar to the problem of the Eleatic School's, and Zeno's, paradoxes. As most of us know Zeno presented a series of mathematical paradoxes that have baffled mathematicians and philosophers until Russell developed his theory of infinite regression (innumerate regression). Prior to Russell both the Greeks, and those who followed, were obliged to leave most discourse on infinities to a simple statement of, "as small as we please and as great as we please." For example, we find the following -- which was considered to be a truism by Socrates rather than a series of paradoxes -- from Zeno (a student of Parmenides, 5th B.C.E.):

1. Dichotomy: There is no motion, for whatever is transformed into motion, it will be required to reach the middle (of the distance) before it reaches the end (and for reaching that half-way point it will have to reach half of the half-way point, and so on *ad infinitum*).

2. The Arrow: Zeno states, every object is either at rest or in motion when it occupies the space equal to its own. That object is in that space now (this instant) and always. The moving arrow, therefore, is at rest (and not moving). This is a paradox.

There are two more paradoxes of Zeno presented in the Jowett's translation of *The Dialogues of Plato* (Vol.2.) The two cases outlined above could not be explained away without the innumerate processes in the nature of motion of physical objects. Unfortunately, this did not allow for division *ad infinitum*. Such sequences which could have a finite sum may come under the sets with innumerate members. According to Socrates, however, Zeno's paradoxes were not directed against the Pythagorean Schools because they dealt with ultimate units.

The Jaina School also dealt with Karma theory through the theory of ultimate units as we have seen already. Even the phases of the

bios are dealt with by a measure in terms of the indivisible-corresponding-sections (*avibhagi-praticchedas*), which calculates the emotions in terms of the tetrads in the ultimate particles of matter bound as *karma paramanus*. These have the configuration (*prkrti*), mass number (*pradesas*), life-time (*sthiti*) and energy-level (*anubhaga*). As for the innumerate number, it plays a role in between the finite and infinite.

Paradoxes of Cantor's Infinite Sets and Jaina Set Theory

Let us have a look at the paradox of Cantor's Set theory when it was in its inception. Hausdorff states, "It is to the undying credit of Georg Cantor that, in the face of conflict, both internal and external against apparent paradoxes, popular prejudices, and philosophical *dicta* -- *infinitum actu non datur* -- there is no actual infinite. Even in the face of doubts that had been raised by the very greatest mathematicians, he dared this step into the realm of the infinite (*Set Theory*, 1962, New York, p.11)."

In 1901 Bertrand Russell discovered that a contradiction could be derived from the axiom of abstraction (which was one of the basis of Cantor's Set theory). He considered the set of all things which have the property of not being members of themselves. The paradox can be related through the barber's paradox in the following story. There is a barber in a village who shaves all those who do not shave themselves. The problem is, who is to shave the barber. Such a set is contradictory to its very existence, but in the Jaina Karma theory the set of indivisible-corresponding-sections of omniscience could have as its constituent member as the set itself. In the physical nature of things we have to set a limit even to the measure of the greatest infinite set.

Russell's paradox is called the logic of the mathematical paradox because it arises from purely mathematical constructions. The barber's paradox may be called the linguistic or the semantical one. Russell's paradox was introduced to show that the obvious, direct axiomatisation of intuitive set theory is inconsistent. The set of all things automatically leads to an infinite set and perhaps to the greatest set. Could this set be a member of itself?

In the Jaina theory of Karma, sets are constructed which have real existence, otherwise the constructs are refuted. Similarly, whenever occasion arises to calculate terms, one gets terms beyond a limit which are avoided as inconsistent. Take for example, the set of the omniscience of all the accomplished souls. This set will have only one value and that will be the omniscience itself. This solves Russell's paradox. However, it was unfortunate for the creator of set theory, Georg Cantor, whose

foundational edifice fell before him. Attempts to revise the foundation of mathematics were soon at hand, and various schools arose in Europe having a school of logistics, school of intuitionism, and a school of formalism. □

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ASTI AND SYĀD IN JAINA THOUGHT

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Prescription of Five Precepts, which are known as *aṇuvratas* or *mahāvratas*, emphasize primarily on the personal conduct and social and societal behavior that is valid irrespective of the time and place. Remarkably the emphasis and the motive for such a conduct is neither based on divine insights nor on not-verifiable knowledge. The Jaina precept of ahimsa is an example of par excellence to cite, as it derives its force from an appreciation of deeper patterns of human behavior and aspirations articulated by ahimsa paramo dharma - "live and let live." It is not just about survival but far beyond the survival instinct.
Interpretations of what is an acceptable level of violence remains a matter of compromises or adjustments in the moral and humane living. Debate about the rationale for such compromises or adjustments sets the intellectual tone for the Jaina doctrine of karma.

To establish *asti* - *what is,* only verifiable knowledge is paramount in Jainism. A prerequisite to have verifiable knowledge therefore is through truthful conduct, impressively implicated in the question of *gaṇaḍharas* to Lord Mahāvīra:

Q. Can *ātmaswarupa* - inner form be known from *gyān* or from *agyān*?

A. Certainly it is not possible without *gyān*, however, to see the whole it is also necessary to know *agyān*.

This insight underlines and highlights not only a useful guidance for personal conduct, but also triggers an impetus for the *anekānta* doctrine which entertains alternatives essential to reduce the level of *agyān*, that is what *is not* known. Logic syllogisms developed for this purpose formalize the thought process for arriving at a valid inference and reduce the level of doubt. This is both critical and important to develop good attitude, to good decision-making, and for evaluating liabilities of the knowledge base. Liabilities in the use of gyān for inference come from not knowing what is not known. This is where

every inference is subject to '*syād.*' As formulated by Mallisena (c. 1290 C.E.) in his *Syādvāda Mañjari* such doubt is intrinsic in any valid inference. While to know certain facts required to develop an (partial) understanding of what *it is - āsti,* it is a case of never know all the necessary facts to elaborate what *it is not - nāsti.* Obviously the 'whole,' alluded to in the answer of Mahāvīra, is the sum total of *is* and *is not*, in the sense of a system that is described by Set Theory.

Logical syllogisms are based on intrinsically rigorous mathematical methods that assure internal consistency and completeness. Sensory system with intrinsic limitations and the linearity of conventional language fragments knowledge, as with the conceptualization of a whole elephant by the knowledge of its body parts not only requires the knowledge of its morphology, habitat, social surrounding, and the course of its personal history and genealogy. More over it is always colored by personal experiences and not necessarily by a set of facts, rules and laws to organize the observable. Hence, the piecing together of the available fragmented information is postulated by the doctrine of anekantavada by Mahāvīra.

What is not known falls in the realm of agyān, and it is clearly distinguishable from the wrong, unverified and contradictory knowledge which could be categorized as mithyāgyān. Later Jain saint scholar philosophers such as Samantabhadra, Siddhasena, Mallavādi and Jinabhadragaṇi articulated and elaborated that *doubt* is worthy of deeper intellectual inquiry with a true value in relation between *mithyāgyān* and *doubt.*

Like *asti* and *nāsti, syād* (doubt) and contradictions have deeper intellectual roots that go back at least 3000 years. The Aristotelian inference syllogisms based on two states - true and false - have been developed in the form of Boolean Algebra, which is the foundation for modern computer science. In an attempt to explore the truth inherent in *doubt*, Prof. G.N. Ramachandran of the Institute of Science in Bangalore India has developed a second and third order Boolean Matrix Algebra (*Mathematical Philosophy Report* #79,1990). This approach applied to understand the deeper structure of *syādvāda* and *saptabhangi* syllogisms provides insights.

In the Indian tradition, the inference schema is illustrated by examples such as the following one:

There is fire on the hill (based on the fact that)
There is smoke on the hill (the rationale is that)

Wherever there is smoke there is fire (is it really true?) as in kitchen
tatha there is fire on the hill (the assertions now hypothesis)

The purpose of this syllogism is to reduce the level of doubt. The past experience of the fire in a kitchen is used to infer a current unknown event. The initial assertion is rationalized by the connective *tathat*, which means 'based on the previous facts it follows.' It is obvious that without such a formal secular syllogism for arriving at a valid inference it would be impossible to coexist, communicate and develop democratic institutions including science.

The Greek syllogism attributed to Aristotle relies on a near-absolute knowledge that 'all humans are mortals. Since 'Socrates is a human,' *tathat* 'Socrates is mortal.' A fact that is not often appreciated; and at least not overtly recognized by Greek syllogism is the acknowledgment of the liabilities intrinsic in the inference. i.e. inference is as good as the knowledge on which it is based. Similarly the possibility that there could be smoke without a fire is left open in *Nyāya*. And therefore it becomes the basis for the principle of *syādvāda*.

Syādvāda is not a figure of speech as meant in the common usage of the word 'perhaps.' nor it is a rhetorical devise. Similarly, the doubt intrinsic in the inference does not come from unverified knowledge, as:

- it cannot be entertained the possibility that what appears to be smoke could be a rain cloud or a dust storm;
- Contradictions where true or false coexist cannot be considered;
- Not concerned with what is 'unknowable, or 'cannot be fathomed.'

Such themes are dealt by mystics who might even surmise that 'knowing nothing is the reason to doubt that one knows everything.' Similarly, syād does not arise from the momentary, probabilistic or average character of what 'it is,' or nor does it relate to fuzzy-logic. In order to identify the logical basis for the origin of doubt, formalisms to express doubt as uncertainty with statistical significance have been developed. Such approaches are of course useless for reconstructing reality from a set of events. Therefore *syād* is the deterministic statement about the doubt intrinsic in an inference based on two (or more) events. It represents the kind of doubt that is always present in any scientific statement and it is necessary for further inquiry to reduce the level of

doubt. For example, 'if only one of the two women is pregnant.' the probability of each being pregnant is 50%; however the assertion that each of them is 50% pregnant is meaningless

The Aristotelian binary logic of 'true' or 'false' (is or is not) has set foundations of Boolean Algebra, which forms the conceptual basis for all computer operations based on the binary states, 0 and 1, as intrinsic in the set theory. For a set of A and B, it can be said 'not A is B' or 'not B is A.' This is because the universe of the A+B set is a closed universe:

> i.e. if we know which one of the two women is pregnant, by implication we also know which one is not.

The usefulness of a syllogism comes from the implication. More situations develop under conditions where all the elements of a set are not defined, or their relationship is governed by nonsymmetry of implication. For example, if P implies Q, it does not necessarily mean Q implies P. Consider the verb 'implies' which is also used as a logical connective. For example, in the inference schema, if smoke always implied fire, and if fire always implied smoke, it will be closed argument where fire is equivalent to smoke, and therefore there is no need for inference.. It will be self evident and redundant statement (tautology). On the other hand, consider the statement from a person who says "what I say is not true." Is it really true or false? This question has baffled Western logicians ever since Aristotle. But according to *syād* principle, it is obviously a contradictory statement and therefore not worth a discourse.

Genesis of doubt in an implication can best be appreciated by the following statement:

> A or B implies C.
> If we know either A or B, we know C.

However, by knowing that C is true, one does not necessarily know A or B except for the fact that at least one of the two is true. If both A and C are true, B can be true or false resulting in the outcome of the ***doubtful*** inference. If true and false states are presented in notation, they may be symbolized in the following form:

> True and false represent (1, 0) and (0, 1) respectively.
> Doubt and Contradiction represent (1, 1) and (0, 0) respectively.

Many such situations may be routinely encountered, and therefore additional and independent knowledge about the system is needed in order to resolve such doubts. Prof. Ramachandran as noted earlier has shown that this representation of doubt has numerous advantages and it can be put to use in computer algorithms. As such the evaluation of truth functionality of three statements about a system can be adequately represented by three bits - true, false and inexpressible - which predict seven states of truth. The eighth state which represents 'neither is, nor is not, nor inexpressible' (0,0,0) is a contradictory and therefore must be left out for the consideration of mystics.

The seven states of truth - the *spatabhangi* example from the *Syādvāda Mañjari* readily illustrates the physical significance of these syllogistic states:

syād	it is	(1,0,0)
...	it is not	(0,0,1)
...	it is inexpressible	(0,1,0)
...	is, is not	(1,0,1)
...	is, is inexpressible	(1,1,0)
...	is not, is inexpressible	(0,1,1)
...	is, is not, is inexpressible	(1,1,1)

In short these secular thoughts represented by notation impressively explicit the Syādvāda and Saptabhangi principle being not only consistent with each other but they are extensions of binary Boolean algebra. Inference syllogisms of higher order are obviously widely accepted for intellectual discourse but their mathematical foundations are not established. Interestingly, these syllogisms have intellectual continuity that emphasizes attempts to develop a secular basis for the elaboration of the deeper forms and inner structures of thought and for the interpretation of content and physical significance. Prof. Ramachandran has shown that the four states resulting from two elements generate foundations for the prepositional logic, and it appears that the eight states resulting from three elements could provide a basis for the predicate logic. □

Mathematical Contents Of Jaina Texts

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Certain mathematical contents in Prakrit texts of the Jainas are explicated in regard to the view of symbolic mathematics employed in the *Karṇāṭaka Vṛtti* (commentary) on the *Gommaṭasāra* and the *Labdhisāra,* as enumerated by Pandit Ṭoḍaramala (c.1721-61 C.E.). This centralizes the karma theory in the Jaina School. The cosmological theory contains both astronomical and geographical mathematics with deeper in approach to the setting of the mathematical background of a model.

Jaina technical terms in Prakrit texts of the *Karanaṇānuyoga* or the *Dravyānuṇyoga*, import mathematical significance - philosophy tinged with mathematics. As Bertrand Russell has in his *Introduction to Mathematical Philosophy,* the Jaina mathematical philosophy appears in the texts for the first time. The *Karṇāṭaka Vṛtti* of the *Gommaṭasāra* carries this mathematical philosophy in detailing the Jaina karma system in symbolic forms, arithmetical, algebraical and geometrical.[1]

Mahāvirācāraya,[2] the author of the *Gaṇitasāra Saṁgraha*, did collect the mathematical material from the Jaina source material and he goes further in stating that whatever else is to be said may be seen in the *Āgama.* Śridharācāraya is still controversial, however, in the history of mathematics.[3]

The *Parikarma*, a commentary work of Kundakundācāraya and the *Jyotiṣapaṭala*[4] of Mahāvirācāraya are not yet available. Also, reported commentary works of celebrated Tumbulūrācāraya and Samantabhadrācāraya are not available. These could have traced and detailed the algebraic symbolism of the *Karṇāṭaka Vṛtti* of Keśava Varṇī.

There are several problems in the history of mathematics and science regarding the source in India, as talents of scholars of Jainism and Prakrit have not been channeled towards at university level. Only recently, university of Bhopal in India has a bold step in this direction by offering a course in the department of religion and culture.

There are problems regarding the origination of and motivation of a paradigm shift in the terminology and usage of symbols. In the Prakrit texts, we find the logical and philosophico-mathematical terms -

as in other philosophies - but the mathematical manipulation through symbols is a peculiarity of the Digambara Jaina School. Similar achievement in southern India appears - perhaps round about the period of Kundakundācārya, when writing of the scripture was in full swing after the compilation of the *Ṣaṭkhaṇḍāgama* and the *Kasāyapādhuda* texts.[5] Round about this period we find certain revolutionary events which speak of the mathematical talents of some genius.

Mathematical Terms, Symbols, and Events Of Early Common Era

Without going into their controversial details, but leaving it up to the scholars to solve the problem of their source on the basis of an indispensable necessity, let us shall relate the events.

Zero in the place value system was needed by the Jaina School. We also find - the place value system employed in the addition of the factor as well as in their subtraction - in the *Karṇāṭaka Vṛtti*. Zero was adapted in the writing of the *Mahābandha* to fill up the gaps and so on.

The Jain calendar records a precession in the *Vedāṁga Jyotiṣa* calendar during this period, and the establishment of Vikrama *Saṁvat* in India. Perhaps this was the era when various texts quoted by Vīrsenācārya were compiled for mathematical imports of the karma philosophy. For example, the *Varangā sūtra*, the *Vedanākṣetravidhāna*, the *Khettaniogaddāra*, the *Pariyama*, the *Kālavihāno*, and so on were some of the mathematical texts which could survive against the time.[6]

The cosmological texts including astronomy and geography, e.g., the *Tiloyapaṇṇattī*, the *Sūryaprajñapti*, the *Candapaṇṇattī*, and so on, did not only depict calendrical details as the *Vedaṁga Jyotisa* but there was also a unified astronomical theory, set in a mathematical universe.[7] When several processes are depicted through a single manipulation it becomes a unified theory which is regarded as simple. The Greeks split it through the epicycles for finer calculations. Einstein gave a unified theory. Now there is an attempt for a theory of everything (TOE) on physics. The Jaina School tried to give such a theory of everything for the biological phenomena through the mathematical theory of karma.[8] The question is whether we should computerise such a theory and prepare files in the software to execute programmes as if happening even in astrology. What could be the results for the benefit of society or a nation? Astronomical programmes will be found to be simpler. Roger Billiard has already computerised the Yuga system of Indian astronomy leaving the Prakrit version of a calendar. Success in building up various programmes in the karma theory will depend on how we are able to form states, inputs and outputs from the mathematical data

furnished in the commentaries of the *Gommaṭasāra* and the *Labdhisāra* through C++ language.

Let us have a look at the mathematical material in these texts which could be helpful in computerisation. The simile and number measures (*upamā* and *saṁkhyā pramāna*) are finite and transfinite cardinals and ordinals of various types of sets (*rāśis*). There are fourteen types of sequences (*dhārās*) in the *Trilokasāra*[9] which locate several types of sets and their measures. Every topic in the karma theory deals with the minimum (*jaghanya*) and the maximum (*utkrstṛṣṭa*) fixing the domains and ranges between which the computer is to work. The eight operations called the *parikarmāṣṭaka* not only deal with the finite quantities but also transfinite quantities as well as the fixed and variable sets. The *trikoṇa yantra* (triangular matrix) can be given several programmes for the variability of the measures of the mass number (*pradeṣas*), configurations (*prakṛtis*), energy levels (*anubhāgas*) and the lifetimes (*sthitis*) of the karma ultimate particles (*paramāṇus*). The *Labdhisāra* depicts these variations in a symbolic way of mathematics.[10] The equations and inequality relations given in this way may pave the way to a more complex manipulation of the problems posed in the modern set-up of the *karmic* data.

Before we give the measurable terms it will not be out of place to suggest that the vast mathematical data could be arranged in a computer file in a graded manner. From the lowest value we go to the largest value in a certain programme and these could be coded in one of the computer's high level languages such as Fortran, Mathematica or C++. The controllable and observable situations are defined in terms of the control (*guṇa*) and reachable (*mārgaṇā*) stations (*sthānas*). Thus the computer could be helpful in showing the time-dependent and time-independent phenomenology of the karma theory of the Prakrit texts referred to above.

We now relate only the terms of the Prakrit texts which denote a measure which could be calculated to give a rough or fine gradation or topology. One should note that a variable measure is given in an algebraic way, set theoretic in approach. Its measure is therefore given between its minimum and maximum values and talents are required to approach a proper and suitable value. This could be approximate also as is found in several places of the *Labdhisāra* or the *Sūryaprajñapti*, *Tiloyapaṇṇattī* or the texts of the *Gaṇitanūyoga*. Surely, this is based on probability.

Datta had collected some terms and he tried to give their interpretation admitting that his attempt was premature.[11] The *Dharvalā*

texts and the *Karṇāṭaka Vṛtti* were not before him. Yet one has to delve deeply into the theory also for showing the origination and motivation of the terminology. He dealt with their definitions and their historical importance. Dr. A.N. Singh also attempted the same thing while he contributed articles on the mathematics of the *Dharvalā* and other texts.[12] However, a study of the heirarchy of various topics is needed for fruitful results and recognition.

In this brief article we give certain mathematical terms out of which the asterisk marked will be those whose measure could be rather ascertained (translated into Hindi from their Prakrit version): *Parikarma* (eight operations -- *pratutpanna, bhāgahāra, varga, vargamula, ghana, ghanmūla, saṅkalita* and *vyutkalita*); *Rāśis* (set-synonyms like *samūha, piṇḍa, puñja, sampāta, abhinna*, etc.); *Jīva rāśi**; *Ajīva rāśi**; *śalākā* (counting rod); *saṁkhyāta**; *asaṁkhyāta**; *Ananta**; *Aṅgula* (finger)*; *Jagaśreni* (world line)*; *Loka**; *Palya**; *Sāgara**; *Muhūrta**; *Antarmuhūrta**; *Samaya**; *Pradeśa**; *Varga**; *Vargaṇā**; *Spardhaka**; *Guṇahāni**; *Nānā Guṇahāni**; *Anyonyābhyasta rāśi**; *Samayaprabaddha**; *Pudgala paramānu rāśi**; *Ākāśa pradeśa rāśi**; *Kāla samaya rāśi**; *Kevalajñāna rāśi**; *Kalpa**; *Āvali**; *Rajju**; *Yojana**; *Kalāsavarna*; *Yāvat-tāvat*; *Addhā*; *Uddhāra*; *Vyavahāra*; *Ardhaccheda*; *Trikaccheda*; *Vargaśalākā*; *Vargita-saṁvargita*; *Vikalpa*; *Bhaṁga*; *Saṁdṛṣṭi*; *Gaṇanā Sthanā*; *Oja* and *Yugma rāśis*; *Guṇaśareṇi*; *Sarvadhana*; *Gaccha*; *Mukha*; *Madhyadhana*; *Ādidhana*; *Uttaradhana*; *Dhārās*; *Alpabahutva*; *Utsedha*; *Dhanuṣa*; *Bāṇa*; *Viṣkambha*; *Kṣetraphala*; and so on.[13]

Various terms of the *Labdhisāra* have been defined by Todaramala in the *Arthasaṁdṛṣṭi Adhikāra* of his *Samyakjñāna Candrikā* commentary. These terms like the *Apakarṣaṇa, Utkarṣaṇa*, etc., give operational details in the theory of karma.

The relations between various entities have been given through several formulae, both in the karma theory and the cosmological theory. These formulae can be seen in a collected form in the project work on the *Labdhisāra* assigned by the Indian National Science Academy.[14] For astronomical formulae, some may be seen in *Jaina Astronomy*, published doctoral thesis of Lishk.[15] For cosmological formulae, one can see the "Mathematics of the *Tiloyapaṇṇattī*."[16]

Concluding Remarks

The appearance of the *Gaṇitasāra Saṁgraha* of Mahāvīrācarya in 1912 gave the first indication of the Jaina School of Mathematics in South India. It was a full book on practical mathematics. He was the

first mathematician in the world to recognise the imaginary qualities. Most of his formulae may be seen in other forms in the Digambara Jaina texts on the karma theory. Formulae given in the commentary of the *Sūryaprajñapti* deserve special attention. The mathematics of the Medieval period may also be seen in the works on astronomy and astrology which still await Hindi and English translation. These may be found in the Digambara and Śvetāmbara *Grantha Bhaṇḍāra.* □

REFERENCES AND ENDNOTES

1. *Gommatasāra* of Namicandra Siddhāntacakavartī, Vols. 1-4, Bhartiyajñana Pitha, New Delhi, 1978-81.
2. *The Ganitasāra saṁgraha of Mahāvīrācarya*, ed. and trans. by L.C. Jain, Sholapur, 1963.
3. A recent short article in the *Gaṇita Bhāratī*, Vol. 9 (1987), numbers 1-4, p. 54-56, by Ganitanand, Ranchi, has appeared on the date of Śrīdhara. His remarks are worth mentioning here. S.B. Dixit (1896) had found a reference to Śrīdhara by name in an old manuscriopt of Mahāvīra's *Gaṇitasāra saṁgraha* (ca. 85), and so put the former before the latter. Royal Asiatic Society, Bombay Rs. 230 of *GSS* also ends with the words (*ABORI*? Vol. 31, p. 268).

The similarity of several rules and of many other features between the works of Śrīdhara and Mahāvīra is accepted by scholars. Both may have drawn from a third and common source which is not known or likely to be known. But most of the scholars considered Mahāvīra as a borrower. (He himself named his work as a "collection:.) The date circa 799 C.E. was assigned to Śrīdhara by N.C. Jain, by equating him to the Jaina author of *Joytirjñānavidhi* (799). To reconcile the salutations "Śivaṁ" and "Jinaṁ" of the different manuscripts, it has been suggested that the same Śrīdhara, after writing mathematical works, may have become a Jaina toward the end of his life.

The above note also gives the opinion of B. Dutta and A.N. Singh that 799 C.E. is the probable date of Śrīdhara. It appears that the common source material for both of the above mathematicians has been the *Kasāyapāhuda* and the *Ṣaṭkhaṇḍāgama* and their commentaries which might have been before them. As the Medieval Jaina writers had been writing Jina and Siva for the same deity, some scribe might have had it changed under certain unknown circumstances. It does not seem possible that Śrīdhara could have availed the opportunity of the Jaina source material as a non-Jaina, and he must have compiled the work as a Jaina. It also seems possible that under certain circumstances he might have adopted Śaivism but whether he wrote two such manuscripts after his conversion is doubtful. Thus, looking into the needs of the Digambara Jaina School of Mathematics in South India for their theory of karma, it seems now most probable that both took help from the same source material of the south, and both were Jainas in the Digambara Jaina Schools of Mathematics. For this purpose of convincing argument one may see the project

work on the *Labdhisāra* of Namicandra Siddhāntacakravartī, Indian National Science Academy, 1984-87, by L.C. Jain.

4. Mention has been made by N.C. Jain while he was at Arrah Jaina Siddhanta Bhavana, and this manuscript is not available now.

5. These texts are in several volumes and have gone out of print. New editions of the former are now coming out in the press. *Ṣaṭkhaṇḍāgama* of Ācārya Puspadanta and Bhūtabali, Books 1-16, Amaroati, Vidisha, 1939-1959. Cf. also *Kasāya Pāhūda* of Gunabhadrācārya, along with the Jayadhavalā commentary of Vīrsenācārya and Jinasenācārya, Vols. 1-13, and the following Mathura, 1944-....

6. For the texts of the Śvetambara Jaina School, cf. the exhaustive article, *The Jaina School of Mathematics*, by B.B. Dutta, Bul. Cal. Math. Soc., Vol. xxi, No. 2, 1929, pp 115-145.

7. For details, see the "Jaina Astronomy" by Dr. S.S. Lishk, (1978), Doctoral Thesis approved by the (Patiala) Punjabi University, 1987, Vidyasagara Publications, Delhi. Cf. also, Jain, L.C., "On the Spiro-Elliptic Motion of the Sun Implicit in the *Tiloyapaṇṇattī*", IJHS, Vol. 13, No. 1, 1978, pp. 42-49.

8. Jain, L.C., *System Theory in the Jaina School of Mathematics*, IJHS, Vol. 14, No. 1, 1979, pp. 29-63.

9. *The Trilokasāra of Nemicandra Siddhāntacakravartī*, Sri Mahāvīrji, 1976. Cf. also Jain, L.C., *Divergent Sequences Locating Transfinite Sets in Trilokasāra*, IJHS, Vol. 12, No. 1, 1977, pp. 57-75.

10. Cf. the project referred to in 4.

11. Cf. the ref. 8.

12. Singh, A.N., *Mathematics of Dhavalā, Ṣaṭkhaṇḍāgama*, book 4 loc., cit., Amaraoti, 1942, pp. i-xxiv. Datta, B.B., and Singh, A.N., *History of Hindu Mathematics*, Bombay, 1962.

13. Cf. ref, 4 for details.

14. Cf. Jain, G.R., *Cosmology Old and New*, Gwalior, 1942.

15. Cf. ref.9.

16. Cf. Jain, L.C., *Tiloyapaṇṇattī ka Ganita, an introduction to the Jambūdīva paṇṇattī Saṁgaho*, Sholapur, 1958, pp. 1-109.

THE GAṆITASĀRA OF THAKKURA PHERU

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As John Scott Deyell points out in his Ph. D. dissertation, Living Without Silver: The Monetary History of Early Medieval India submitted to the University of Wisconsin-Madison 1982, the century rulers of Delhi had employed bankers in the minting operations. It is well known that during the period much of the economic activity in the Gujrat-Rajasthan-Delhi region was controlled by the Jains. The members of the Śrīmāli caste of the Jainas were specialized in minting and money exchange. Among these members of the Śrīmālis, Thakkura Pheru stands out as a writer on a wide range of scientific subjects in popular speech. He wrote six scientific works:

1. *Vāstusāra* on architecture and iconography, 19 September, 1315.
2. *Jyotiṣāra* on astrology and astronomy, 1315.
3. *Ratnaparīkṣa* on gemology, 1315.
4. *Gaṇitsāra* on arithmetic, 1318.
5. *Dhātutpatti* on metallurgy and perfumery trade, and
6. *Dravyaparīkṣa* on assay and money exchange, 1318.

All the works are published in the Thakkura-Pheru-Voiracita-Ratnaparikṣādi-Saptagraṇtasamgrha, ed. Munivijaya, Jodhpur 1961.

About the Author

Pheru was born sometime around 1270 C.E. at a place called Kannana which is situated in the modern state of Harayana. Kannana was not only far from the then capital of Delhi but it was the a Jain center of pilgrimage for the Jains. Pheru's father Canda was a prosperous banker, and he had the court title of Thakkura, which suggests that he may have been associated with the treasury at Delhi. Pheru's grandfather Kaliya or Kalasa, although a prosperous banker, did not possess the Thakkura title. The family belonged to the Jaina Kharatara *gaccha*.

Pheru appears to have presumably completed his formal education in 1291 C.E. and during this year he composed the *Karataragaccha-Yugapradhāna-Catupādika*, an eulogy to his Pontiffs.

Later, he joined the treasury of Alauddin Khiji at Delhi, and wrote the *Ratnaparīkṣā*, a manual on gemology, for the instruction of his son Hemapāla in 1315 C.E. In the same year, he wrote the *Jyotiṣāra* and the *Vāstusāra*. During the time of Qutubddin Mubarak Shah, he wrote the *Dravyaparīkṣa* in 1318 C.E. He held the position(s) successfully during the successive Sultāns - Alauddin (1296-1316), Shihabuddin Umar (1316), Qutubddin (1316-1320) and Ghiyasuddin Tughluq (1320-1325).

The Work Gaṇitasāra

The text is not dated but must have been written before 1318 C.E. It is more innovative and not so much in the theoretical portions but in the application of arithmetical rules to a wide range of areas. It is common place to say that arithmetic is one of the most practical sciences, its rules being employed by traders, masons, carpenters, tax-collectors and the like for the calculations connected with their professions. The units of measurement and the examples to illustrate arithmetical rules given in the text throw a flood of light on the economic and social conditions of the period.

In the section of solid geometry, Pheru gives the rules for the volumes of domes (*gonaṁṭa*), square and circular towers with spiral stairways in the middle (*pāyaseva*), towers with fluted columns (*munāraya*), niches (*tāka*), staircases (*sopāna*), bridges (*pulabamdha*) and so on (III.74.86). The purpose of such rules is to enable the chief mason to calculate the number of bricks or stones needed for the these constructions. Accordingly, Pheru points that to do this calculation more exactly one should first calculate the total volume of the wall-space, subtract from this the volume of the space occupied by the doors and windows and reduce the remainder by the three-twentieths, the latter being the volume of the mortar (III.70-71). The result when divide by the volume of a single brick yields the number of bricks.

The *munāraya* is like a circular tower with a spiral stairway in the middle, as far as the inside is concerned. But the difference is the wall contains half triangles and half circles (III.80). The meaning of the cryptic last sentence is that in a horizontal cross-section of the *munāraya*, the outer circumference consists of alternate triangles and semi-circles. It should be remembered that about a hundred years before this time, Qutubddin Aibak built the Qutab Minar in Delhi and that Alauddin himself started constructing another tower twice as high. Now, the lower story of the Qutab Minar consists of alternately angular and circular

columns, and it is clear that Pheru is referring here to such a tower with fluted columns.

In another section dealing with cloth (IV.i.18-37), Pheru mentions different kinds of silk, woolen and cotton materials, the rate of shrinkage or loss in washing, cutting and sewing, and the area of cloth needed to make various types of tents. There is a last section (IV.iii.1-17) listing the average yields of grains, pulses, etc. per *bīghā*, the average yield of mollasses and brown sugar per maund of sugarcane, the amount of clarified butter that can be obtained from cow's and buffalo's milk and so on. His rule of converting Vikrama dates into Hirji dates and vice versa (IV.i.17) which is probably the first such rule to be formulated in India. Though all these are not germane to arithmetic as such but Pheru is adapting arithmetic to suit the needs of a variety of professions. □

This is a condensed part of the paper "Thakkura Pheru and the Popularization of Science in India" in *Jainthology*, Ed. Ganesh Lalwani, Jain Bhavwan, Calcutta 1991.

A Glimpse at the Geometry of Certain Figures

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There was a noticeable achievement in scientific and quasi-scientific compositions during the period of ninth through fifteenth centuries C.E. The paper discusses geometry figures of polygon, ellipse, circular segment and spherical segment presented during this period.

The Polygon

1.1 The *Līlāvatī* (*LV*) of Bhāskara II (b.1114 C.E.) (1, Part II, pp.206-208], see also [5]) prescribes the following rule for finding the length of a side of a regular polygon inscribed in a circle:

त्रिद्व्यङ्काग्निनथश्चन्द्रैः त्रिबाणष्ट्युगाथिः।
वेदाग्निबाणखाश्वैश्य खखाश्राश्वरसेः क्रमात् ।।धएघ।।
बाणेषुनखबाणोश्च द्विद्विनन्देषुसागरैः।
कुरामदशवेदैश्च वृतव्यासे समाहते ।।धएछ।।
खखखाश्वार्क सँथके लथ्यन्ते क्रमशो थुजाः।
वृतान्तरलपूर्वाणाँ नवारलान्तँ पृथक्पृथक्।।धएष।।

(*Kṣetravyavahāra of LV*)

This can be read to mean (cf.[5]): Multiply the diameter of the (given) circle, in order, by (the coefficients) 103923, 84853, 70534, 60000, 52055, 45922, and 41031. On dividing (each of the products just obtained) by 120000, there are obtained the sides respectively of the (equilateral) triangle to the (regular) nonagon (inscribed in the circle) separately.

That is, the length l_n of a side of regular n-gon is obtained by the formula:

(1.11) $l_n = (d/120000)\, k_n$; (d being the diameter of a circle)
where seven coefficients k_n $(3 \leq n \leq 9)$ are as stated in the rule.

Gupta [5] infers that coefficients k_n surely indicate the sides of regular polygons inscribed in a circle of a radius 60000 as $k_n = l_n$ for d = 120000.

Commentator Gaṇeśa (ca. 1545 C.E.) (cf.[5]), however, for the first time tries to give the rationale of k_n by two methods. The first method, based on the table of sines, is far from being satisfactory. The second method which employs the (so called) Pythagoras theorem, discusses only for n = 3,4,6,8, but refutes that the same technique cannot be applied for other cases. From Figure 1 (see below),

(1.12) $l_n = d \sin \pi/n$; (d =2R).

Comparing (1.11) and (1.12),

(1.1.3) $k_n = 120000 \sin \pi /n$.

Comparison of modern values of k_n's ($3 \leq n \leq 9$) with those of *LV* and *Kriyākramakarī* (= KKK) (see[5]) is given in the following chart:

k_n	Modern Calculation	*LV*	*KKK*
k_3	103923	103923	103922 (103923)
k_4	84853	84853	84853
k_5	70534	70534	70534
k_6	60000	60000	60000
k_7	52066	52055	52067
k_8	45922	45922	45922
k_9	41042	41031	41043

1.2 For the area of a regular polygon, Utpala (ca. 10th century C.E.) in his commentary [3] (see also [9]) on Varāhamihira's *Bṛhatsaṁhitā* gives the following rule:

दिर्न्यस्य परिधेर्वर्गमेकस्मादश्रिजार्धितात्।
लब्धं सँशोध्य परतो थक्त्वा द्वादशथिः फलम्। ‖52.73‖

This means: Put the square of the perimeter (*paridhi*) at two places. At one place divide by (square of) half the number of sides. Subtract the quotient (so obtained) from the other (square of the perimeter and (then) divide by twelve. The area (*phalam*) is:

$A_n = [p^2 - p^2/(n/2)^2]/12$
or
$A_n = l_n (n^2 - 4)/12; (p = l_n)$

Whereas the *Gaṇitasārasaṅgraha* (= *GSS*) of Mahāvīra (ca. 850 C.E.) regarding the area of a general plane polygon describes the relation ([15]), see also [10]) in the following:

रज्जवर्धकृति त्र्यँशो बाहुविथक्तो निरेकबाहुगुणः।
स्वेपामभ्रवतां फलँ हि विम्बान्तरे चतुर्थाँशः||V||.39||

This can be translated as

(1.22) $A_n = (s^2/3) (n - 1)/n$

wherein s = (sum of the sides' length)/2.

For a regular polygon, $s = (nl_n)/2$, and therefore from (1.22) [12]

(1.23) $A_n = l_n^2(n - 1)n/12$

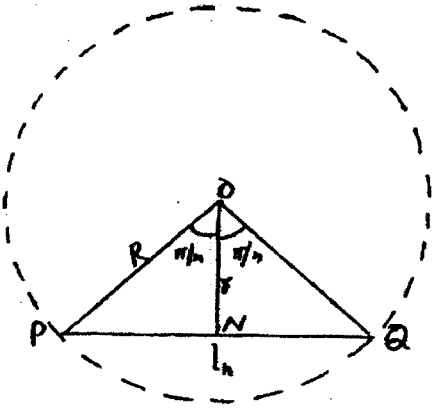
The same rule (1.23) in equivalent form is also found in *Gaṇitakaumudī* (= GK) (c.1356 C.E.) of Nārāyaṇa Paṇḍita (cd.[10]).

We know from the modern analysis that the exact area (which we shall denote byA_{0n}) of a regular polygon is governed by (referring figure 1)

$A_{on} = nl_nr/2$, i.e.

(1.24) $A_{on} = (n/4)l^2_n \cot \pi /n$

Figure 1. Regular Polygon of n-sides



The degree of accuracy of A_n given by (1.21) may be accessed by calculating the percentage error E_n as:

$$E_n = \left[\frac{A_n}{A_{on}} - 1 \right] \times 100 = \left[\frac{n^2 - 4}{3n} .\tan (\pi/n - 1] \times 100. \right.$$

Table 1A show the percentage error if one uses the formula (1.21) for different values of n.

Table 1A

$E_3 = 3.77$	$E_{10} = 3.97$
$E_4 = 0$	$E_{20} = 4.53$
$E_5 = 1.72$	$E_{30} = 4.64$
$E_6 = 2.64$	$E_{00} = 4.72$

The applicability of A_n given by (1.23) may also be estimated by computing the percentage of error E_n below (see Table 1B)

$$E_n = \left[\frac{n - 1}{3} .\tan (\pi/n - 1] \times 100. \right.$$

Table 1B

$E_3 = 15.47$	$E_{10} = -2.52$
$E_4 = 0$	$E_{20} = 0.31$
$E_5 = 3.13$	$E_{30} = 1.60$
$E_6 = 3.77$	$E_{00} = 4.72$

Conclusion

We can infer that the results (1.21) and (1.23) give the exact area in the case of a regular 4-gon (i.e., a square) only and approximate ones for all other regular polygons. Moreover, the amount of inexactness is the same for both results in case the sides are large enough. We remark that (1.23) is fairly applicable when the number of sides is near about 20.

The Ellipse

Datta and Singh ([2], refer also Gupta [11]) find the word *viṣamacakaravāla* (unequal circle) applied for ellipse in the Jain text *Sūryaprajñāpti* (ca. 500 B.C.E.), and Gupta [op.cit.] as *parimaṇḍala* in both Buddhist text the *Dhammasaṅgaṇi* (ca. 400 B.C.E.) and the Jaina text the *Bhagvatīsūtra* (ca. 350 B.C.E.). Mahāvīra in his *GSS* uses the term *āyatavṛtta* (elongated circle) for the same [11]. For an accurate (*sūkṣma*) values of perimeter P and area A of an ellipse, the *GSS* states the following rule ([15]], see also [4]):

व्यासकृतिषड्गुणिता द्विसगुणायामकृतियुता पदँ परिधिः।
व्यास चतुर्थगिगुणश्चायतवृत्तस्य सूक्ष्मफलम्।।V||.63||

This means (cf. [4]): (The square-root of) the sum of six times the square of the breadth and the square of double the length is the perimeter. (That perimeter) multiplied by a fourth-part of the breadth is the accurate area of the elongated circle.

This is the equivalent to

(2.1) $P = 2\,(4a^2 = 6b^2)^{1/2}$

(2.2) $A = b(4a^2 = 6b^2)^{1/2}$

where a and b are semi-major and semi-minor axes respectively. Using the modern, the exact area is found to be

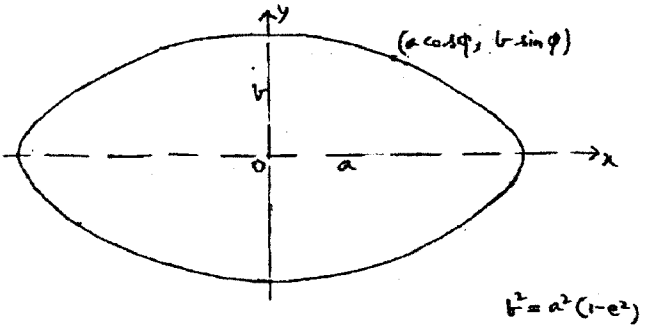
(2.3) $\Lambda_O = \pi ab$

and the exact perimeter

$$P_o = 4\int_0^{\pi/2} (a^2 \sin^2\phi + b^2 \cos^2\phi)^{1/2}\, d\phi \quad \text{or}$$

$$(2.4)\ P_0 = 2a\left[1 - \frac{e^2}{4} - \sum_{n=2}^{\infty} \frac{1^2\, 3^2\, 5^2 \ldots\ldots (2n-3)^2}{2^2\, 4^2\, 6^2 \ldots\ldots\ldots (2n)^2} (2n-1)e^{2n}\right]$$

Figure 2: Ellipse



For very fine approximations of perimeter of ellipse by Ramanujan, refer [11]. The extent to which the perimeter given by (2.1) is valid may very well by judged by computing its percentage of error E_e (see Table 2A):

$$E_e = \left[\frac{P}{P_o} - 1\right] \times 100 = \left[\frac{\sqrt{10 - 6e^2}}{\pi k'} - 1\right] \times 100$$

$$\text{Where } k' = 1 - \frac{e^2}{4} - \frac{3e^4}{4^3} - \frac{5e^6}{4^4} - \frac{175e^8}{4^7} - \cdots\cdots$$

$$\text{Or } E_e > \left[\frac{\sqrt{10 - 6e^2}}{\pi k} - 1\right] \times 100;\quad k = 1 - \frac{e^2}{4} - \frac{3e^4}{64}.$$

Table 2A

$E_{0.1} > 0.36$	$E_{0.6} > -0.97$
$E_{0.2} > 0.48$	$E_{0.7} > -2.78$
$E_{0.3} > -0.10$	$E_{0.8} > -3.66$
$E_{0.4} > 1.44$	$E_{0.9} > -6.28$
$E_{0.5} > -0.21$	

The degree of accuracy of an area specified by (2.2) may also be easily extracted from its percentage error E_e calculated below (see Table 2B)

$$E_e = \left[\frac{A}{\rho} - 1\right] \times 100 = \left[\frac{\sqrt{10 - 6e^2}}{\pi} - 1\right] \times 100$$

Table 2B

$E_{0.1} = 0.36$	$E_{0.6} = -10.87$
$E_{0.2} = 0.56$	$E_{0.7} = -15.42$
$E_{0.3} = 2.10$	$E_{0.8} = -21.0$
$E_{0.4} = -4.29$	$E_{0.9} = -27.83$
$E_{0.5} = -7.20$	

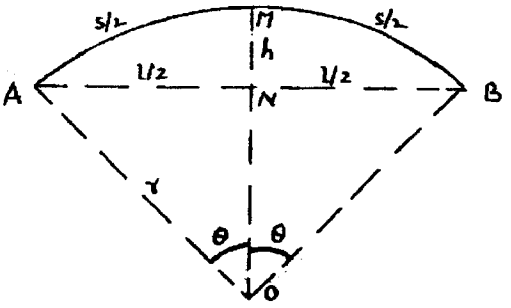
It may be concluded that it is remarkable that the formula (2.1) is applicable for e near 0.1-0.3, 0.5, 0.6 and (2.2) for e in the range 0.1 and 0.2 including vicinities.

The Circular Segment

3.1 Consider a circular arc AB of radius r subtending an angle 20 at the centre. See Figure 3.

The formula (3.11) $s = [l^2 + (\pi^2 - 4)h2]1/2$ with $\pi = 3$ is found in the *GSS* as a practical rule ([6] and [8]). Most Jaina works including the *GSS* use this formula explicitly with $x = \sqrt{10}$

Figure 3: Circular Arc



The validity estimation of (3.11) may be easily inferred from its percentage error Ee (refer Table 3.1):

$$E_\theta = \left[\frac{s}{s_0} - 1\right] \times 100 = \left[\frac{\left[4\sin^2\theta + (\pi^2 - 4)(1 - \cos\theta)^2\right]^{1/2}}{2\theta} - 1\right] \times 100$$

wherein l =2r sinŎ, h = r(1 -cosŎ), s_0 = 2rŎ.

Table 3.1

$E_0 = 0$	$E_{105} = -1.50$
$E_{15} = 0.11$	$E_{120} = -3.89$
$E_{30} = 0.40$	$E_{135} = -7.25$
$E_{45} = 0.73$	$E_{150} = -11.57$
$E_{60} = 0.92$	$E_{165} = -16.82$
$E_{75} = 0.47$	$E_{180} = -22.88$
$E_{90} = 0$	

Evidently (3.11) is technically applicable of $0° < \check{O} \leq 90°$. It is worth noticing that this formula is exact when Ŏ is 90°.

3.2 *Tiloyapaṇṇatti* of Yativṛṣabha (fl. 473-609 C.E.) uses the following formula [8]:

(3.21) $A = [10(lh/4)^2]^{1/2}$

to find the area of various regions of Jambūdvīpa (Jambu Island). These regions are of the circular segment form.

The formula (3.21) in its general form may be considered as

(3.22) $A = \pi lh/4 = \pi r^2 \sin\check{O} (1 - \cos\check{O})/2.$

However, the true area of A_0 is given by the following:

A_0 = Area of the sector OAMB - Area of the triangle OAB.

That is (3.23) $A_0 = r^2 (\check{O} - \sin\check{O} \cos\check{O}).$

The percentage of error E_e will furnish the degree of accuracy of the area given by (3.22) (see Table 3.2A):

$$E_0 = \left[\frac{A}{A_0} - 1\right] \times 100 = \left[\frac{\pi \sin\theta(1 - \cos\theta)}{2(\theta - \sin\theta\cos\theta)} - 1\right] \times 100.$$

Table 3.2A

$E_0 = 0$	$E_{105} = -8.29$
$E_{15} = 17.80$	$E_{120} = -19.26$
$E_{30} = 16.11$	$E_{135} = -13.61$
$E_{45} = 13.98$	$E_{150} = -51.96$
$E_{60} = 10.74$	$E_{165} = -80.53$
$E_{75} = 6.19$	$E_{180} = -100.00$
$E_{90} = 0$	

Notice that he following formula for the area of a circular segment

(3.24) $A = \pi (1 + h)h/6$

appears in various texts, viz., the *Gaṇitapañcaviṁśī,* the *Triśstikā* of Śrīdhara (ca. 8th century C.E.), the GSS, the *Mahāsiddhānta* of Āryabhaṭa II (ca. 950 C.E.), the *Gaṇitasāra of* Ṭhakkura Pherū (ca. 1315 C.E.) and the *Pañcaviṁśatikā* (ca. 1428 C.E.) (see [13]). In polar form, the formula (3.24) may be written as

(3.35) $A = \pi r^2 (2\sin\check{O} + 1 - \cos\check{O}) (1 - \cos\check{O})/6,$

and that the corresponding percentage error is given by the following formula (see Table 3.2B)

$$E_\theta = \left[\frac{\pi(2\sin\theta + 1 - \cos\theta)(1-\cos\theta)}{6(\theta - \sin\theta\cos\theta)} - 1\right] \times 100$$

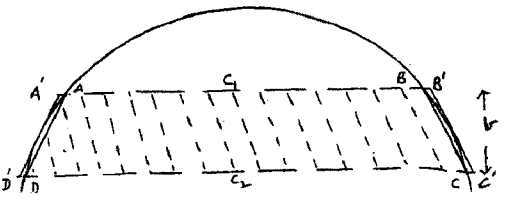
Table 3.2B

$E_0 = 0$	$E_{105} = 0.98$
$E_{15} = -16.60$	$E_{120} = 0.44$
$E_{30} = -12.18$	$E_{135} = -2.32$
$E_{45} = -8.27$	$E_{150} = -8.22$
$E_{60} = -4.86$	$E_{165} = -18.32$
$E_{75} = -2.04$	$E_{180} = -33.33$
$E_{90} = 0$	

It may be conclusion that the formulae (3.22) and (3.24) furnish the true area only when $\breve{O} = 90°$. The formula (3.22) is practically applicable when $\breve{O}$ is near approximately 0° and 90° whereas (3.24) is technically suitable in and near about $0° < \breve{O} \leq 120°$.

3.3 The *Bṛhatkṣetrasamāsa (cf.[7])* of Jinabhadra Gaṇi (6th century C.E.) uses the following formulae to find the area of the zone of the circle (see Figure 4 also).

Figure 4: Zone of the Circle



(3.31) $A_1 = \frac{1}{2}(c_1 + c_2)b.$

Clearly it represents the area of the inscribed isosceles trapezium ABCD and is less than the true area and so it can be treated as a rough approximation.

(3.32) $A_2 = [½ (c_1^2 + c_2^2]^{½}b.$

The area A_2 is less than the circumscribed isosceles trapezium $A^1B^1C^1D^1$. for a plausible derivation of (3.32), refer [7].

The Spherical Segment

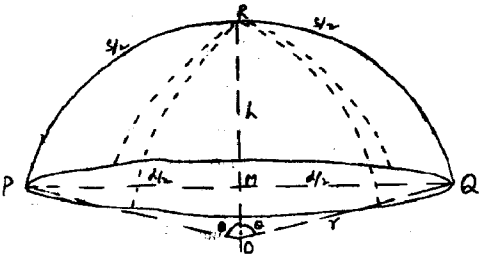
The *GSS* ([14], see also [8]) enunciates the following rule to find the surface area of a spherical segment:

परिधेश्च चतुर्थगो विषकम्थगुणः सविद्धि गणितफलम् ||V||.25||

This means (cf. [8]): Know that one fourth of the circumference multiplied by *viṣkambha* gives the area (of the concave and convex surfaces). According to Gupta [8], this can be interpreted as the following:

A = (p/4) (curvilinear breadth) = π ds/4
or
(4.1) $A = \pi r^2$ Ŏ sinŎ
using $p = \pi d$, $d = 2r \sin Ŏ$ and $s = 2rŎ$.
However, the true surface area is
(4.2) $A_0 = 2\pi rh = 2\pi r^2 (1 - \cos Ŏ)$
wherein $h = r(1 - \cos Ŏ)$.

Figure 5: Spherical Segment



The degree of validity of the surface area given by (4.1) may be estimated just by finding the percentage of error E_0 (refer Table 4):

$$E_\theta = \left[\frac{A}{A_0} - 1\right] \times 100 = \left[\frac{\theta \sin\theta}{2(1 - \cos\theta)} - 1\right] \times 100$$

Table 4

$E_0 = 0$	$E_{105} = -29.69$
$E_{15} = -0.44$	$E_{120} = -39.54$
$E_{30} = -2.30$	$E_{135} = -51.20$
$E_{45} = -5.19$	$E_{150} = -56.37$
$E_{60} = -9.31$	$E_{165} = -81.04$
$E_{75} = -14.71$	$E_{180} = -100.00$
$E_{90} = -21.46$	

Notice that (4.1) is useful for $\check{O}$ up to and near 15°. □

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Zero In Place Value System of Jain Mathematics

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In view of the knowledge derived from Babylonian mathematical cuneiform texts by Neugebauer, Sachs and others, and the claims of Needham and Ling in the "Science and Civilisation in China," various options have been set forth regarding the use of zero and its adoption in the place value system. In India, such volumes as the *Dhavalā*, the *Jayadhavalā*, the *Jīvatattvapradīpikhā Karṇātaka* and other commentaries as well as the *Tiloyapannatti* have brought forth rich mathematico-symbolic material. The karma theory points towards the necessity of a place value system in the *Purvagata* texts - parts of the *Agrāyaṇī* and the *Jñanapravāda* - current in the Digambara tradition. They are required to be studied for further opinions and investigation regarding origin of the zero and its use in place value systems. The model for astronomy in India had to do with the geometry of a circle and a straight line and their extension to cosmological details. The study of Jaina karma system as cybernetics in an abstract form is a unified theory of bios and matter with a mutual dynamic interactions, and the Jaina School of algebra and arithmetic was the outcome of the unique experiences and examinations thereof.

Recent INSA project study on the Jaina School of Mathematics has clearly identified two schools: the Digambara and the Śvetāmbara. The former held proficiency in the mathematico-symbolic theory of karma whereas the latter seems to tend towards the preservation of religious tenets, astronomy and astrology. The paper highlights this fact that more symbolic details on the use of zero are available in the Digambara School than in the Śvetāmbara tradition, contarary to the findings of Datta, Singh and Kapadia.

Introduction

The study of the mathematical cuneiform texts of the old Babylonian period (1600 B. C. E.) has shown their system of numerationis to be based on place-value notation, a sezagesimal scale. Neugebauer opines that it was transmitted to the Greeks and then to the Indians who contributed to the final step by using the place-value

notation for the smaller decimal units.[1] He also finds that the Arabic form for zero symbol (a little circle with a bar over it and related forms) is simply taken from the Greek astronomical manuscripts, as recognised by F. Woepoke in 1863.[2] However, he felt that no definite answer could be given to the question when the zero sign was introduced in Babylonian mathematics, which did not exist before 1500 B.C.E., but was in full use from 300 B.C. E. onwards.[3]

Needham and Ling find that in the far east, the Shang Oracle bone numeral forms (14001100 B.C.E.) and the method of writing numbers with them were based on the decimal place-value ideas continued in the rod numerals.[4] This suggests the possibility of the discovery of zero in south-east Asia where Indian culture "met the southern zone of the culture of the Chinese." They also opine that ancient systems point to a date well before +500 for the development of place-value and the zero concept, it was never entirely convincing due to uncertain chronology of Indian history and the difficulty of dating literary and epigraphical evidence. Apart from this, there has been an emphasis on the "emptiness" of Tao mysticism and the "void" of Indian philosophy whicg are said to have contributed to the invention of a symbol for *śūnya* (zero).[5] To this, the Jaina annihilation of a karma existential matrix -- perpetuating transmigrations that lead to a zero or nil existence of the world culminating in an eternal life of bliss and unending power -- may be added.[6] In this regard, the role of zero and decimal place-value system may be presented from the recently-published Digambara Jaina texts like the *Tiloyapaṇṇatti*,[7] the *Chakkaṇdāgama*,[8] the *Kasāyapāhuda*,[9] and the *Mahābandha*[10] of Bhūtabali. Their summary texts by Nemicandracārya in his works - the *Trilokasāra*, the *Gommaṭasāra*, the *Libdhisāra*,[11] and the *Kṣapaṇasāra* along with its commentaries by Mādhvacandra Traividya, Keśavavarṇī and Ṭoḍaramala may also be referred when required. These texts give an elaborate exposition of a systematic development of mathematical models for geography, astronomy, cosmogony, cosmology and karma theory; as well display the need for systems to express large numbers and set-theoretic manoeuvre.

The INSA research projects -- "*Labhdisāra* of Nemicandra Siddhānta Cakravarti" (1984-87) and "Mathematical Contents in the Digambara Jaina texts of the *Karaṇānuyoga* Group (1992-96) conducted at Indian National Science Academy have shed more light on the use of zero in the form of a circular symbol and its application in the place-value system of the Digambara Jaina School of Mathematics.[12] Round about 1935, some stray material on zero and its use in the decimal

place-value system were located in the form of big numbers from Śvetambara texts by Datta,[13] Singh[14] and Kapadia.[15] Word-symbol notation was also used by Jinabhadragaṇi.

So far as the Digambara Jaina texts are concerned, Datta and Kapida found the zero and its use in decimal place in greater strength in the *Gommaṭasāra* of Nemicandra Siddhanta Cakravarti. The *Gaṇitasārasaṅgraha* of Mahāvirācārya (9th century C.E.) had already been published in 1912 with its use of zero not only in place value but also in *Parikarma*. A few years later, Singh took up the challenging task of exploring the mathematical contents of the third and fourth volumes of the *Dhavala* Commentary on the *Satkhandāgama*.[16] This consisted of the use of zero in a variety of expressions in decimal place value.[17] Later, L.C. Jain located similar material in the *Tiloyapaṇṇatti* (c. 5th century Prakrit text on the information about the three worlds or universes).[18] He found additional material on the same topic in the ASG and ASL chapters in the *Samyak Jñana Candrika* - in the Ḍhumdhari dialect of Rajasthan - a detailed commentary on the *Gommaṭasāra* and the *Labdhisāra*, by Ṭoḍaramalla.[19]

Use of Zero in the Digambara Jaina School

In the above mentioned texts and commentaries the zero symbol (a circle) has been used as follows:

1. Represents through vowels and in the *Kaṭapayādi* system.[20]
2. Stands for a negative sign.[21]
3. Stands for one-sensed and two sensed bios and so on.[22]
   For example, a three-sensed bio is denoted by 000.
4. Stands for a void - *agrhita* stage in which a bio does not assimilate karma particles in *pudgala parivartana* (specific material change).[23]
5. Stands for filling up the gaps.[24] This appears in the writing of the *Mahabandha Sutras* implying a bridge. Ṭoḍaramalla has also expressed such a use in bridging the intervening *iṣekas* (cells) in a column of lifetime (*sthiti*) structure, denoted in the following fashion:

```
    9
    0
    0
5   1   2
```

The triangular karma life - a number of karma particles with certain energy level in a specific configuration (*prakrti*) in every *niṣeka* - is a complicated structure.

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6. Increased through in number through a lower script.[25] For example,

65 000 is written as 65 0
3.

The above symbolism is at its supreme in the *Karṇātaka Vrtti* of the *Gommaṭasāra*, compiled by Kesava Varṇī (c. 13th century C.E.).[26]

(i) The use of zero as a place value might have preceded or followed the use of other numerals as place values, in order to express a big number, needed in their mathematical philosophy of karma, replacing a cumbersome method. For example, in the *Śaṭkhandāgama*[27] the number of developed human beings is stated as lying between 2^{26} and 2^{27} which is also stated to lie between $((10)^7)^3$ and $((10)^7)^4$ or between *Koḍākoḍākoḍi* and *Koḍākoḍākoḍākoḍi*. Singh has already remarked[28] how Vīrasenācārya - the author of the *Dhavalā* - quotes verses representing various styles to express big numbers, apart from the place value with different directional moves.[29]

(ii) It is interesting to see how words and numbers were joined to express a number 61, 97, 88, 46, 66, 81, 64, 16, 20, 00, 00, 000 in verse form quoted by Vīrasenācārya from an earlier ancient work.

(iii) In the *Tiloyapaṇṇatti* (c. 5th century C.E.) the decimal place value notation has been usual as from right to left.[30]

(iv) In the same text, the denominated time units are developed, stopping for a while at "*acalatma*" which denotes $(84)^{31}$ $(10)^{90}$ years.[31] This number has been expressed as (84)/31/90 where 31 stands for the product of 84 to be done 31 times into itself; and 90 stands for zeroes to be placed after the result of the product as place value.

(v) An important application of the place value for subtraction of some desired factors among a set of factors from their product has been used in the *Tiloyapannatti*,[32] the commentaries of the *Gommaṭasāra*[33] and the *Labdhisāra*,[34] most frequently for expressing numbers arriving naturally as results in dealing with the systematic karma theory.

Decimal and Notational Places in DVL

The *DVL*, Books 3 and 4, contain good material on decimal places. The source book *CKG* contains the following words which signify the decimal places (c. 2nd century C.E.):

Word	Meaning	Verse	Book (volume)	Remarks
coddasa	fourteen	2	*CKG*. 1	*coda* for four, *dasa* for ten
dasa	ten	p. 236	2	projected in *DVL* from earlier source
suṇṇa	zero	*DVL* p. 36	3	
sahassa	one thousand	*DVL* p. 37		
terasa	thirteen	*DVL* p. 40		*te* for three, *dasa* or *rasa* for ten
aṭṭhattīsa	thirty-eight	p. 34		*aṭṭha* for eight, *tīsa* for thirty
saya	hundred	p. 36		
tehattari	seventy- three	p. 36		*te* for three, *hattari* for seventy
ekkavisa	twenty-one	*DVL*, p. 67		*ekka* for one, *visa* for twenty
ṇaudi	ninety	*DVL*, p. 67		
solasa	sixteen	p. 37		*so* for six, *lasa* for ten
aḍyālaṁ	forty-eight	p. 39		*aḍ* for eight, *yālaṁ* for forty
vārasa	twelve	p. 40		*vā* for two, *rasa* for ten
koḍi	crore	7		
lakkha	lac	*DVL*, p. 89		

[Note: Here 59398206 is written in words as *pañca koḍio teṇauḍilakkha aṭṭhinaudi sahassā chauttaraṁ visadaṁca*. This is from left to right in decimals and numerals.]

cauvanna	fifty-four	9	3	*cau* for four, *vanna* for fifty
saṭṭhi	sixty	*DVL*, p. 92		
sada	hundred	9		
culasīī	eighty-four	p. 43		*cul* for four, *asīī* for eighty

[Note: On p. 52, the number 4666664 is described beginning with sixty-four, then towards left six hundred, then sixty-six thousand, then sixty-six hundred thousand, then four crore:

causatthi chacca sayā chāsaṭṭhi sahassa cava parimānaṁ /
chāsatthisaya sahassā koḍi caukkaṁ pamattanaṁ / / 52 / /]

[Note: On p. 56 the style is quite different for writing 69999996, which is spoken as six in the beginning, six in the end and nine six times in the middle:

"chakkādi chakkaṁtā channaramajjhā ya saṁjadā savve /"]
sattakodisayā seven hundred crore p. 68
$7X(10)^9$

[Note: In verse 45 of *CKG*, the number of developed illusion visioned human beings is given as above ***koḍākoḍākoḍie*** and below ***koḍākoḍākoḍākoḍie***; i.e., above (10) and below (10)

"manusapajjattesu micchāiṭṭhi davva pamāneṇa kevadiyā
koḍākoḍākoḍie uvari koḍākoḍākoḍākoḍie heṭṭhado//"]

From the above it appears that the decimal places had to do with the vertical line or horizontal line and zero. It also appears that if *dasa*, a word for ten, was abbreviated from *daṁda* and *suṇṇa*, where *daṁda* stands for a rod and *suṇṇa* for a zero. Similarly, *soda* appears for one hundred, abbreviated from *sadasa*, or a *suṇṇa* added to *dasa*. Similarly, *sahasa*, a word for one thousand, appears from *ha* as *haṭṭha* for a rod and three "sss" for three zeros. Further, *sada sahassa, dasa sahassa* could be interpreted for their choice of denomination for one *lakh* and 10000. One *koḍī* (a crore) also comes under *dasa soda sahassa*. This could possibly be thought of as their entry into the realms of a decimal place value system.

Notational Places in TPT

The *TPT*describes two fundamental instant sets, the *Palya* and the *Sāgara*, needed to give measures of various exisential sets, as simile construction sets. In the process of evaluating the *Palya*[35] the number of *Palya*-hair (palya-roma) is a number given by (41345263030820317749512192) $(10)^{18}$ expressed as a product of 19/24 and the following three rows:

SO/96/500/8/8/8/8/8/8/8/8/
SO/96/500/8/8/8/8/8/8/8/8/
SO/96/500/8/8/8/8/8/8/8/8/

where SO stands for three zeros, to be placed on the right after the product. The above number is expressed in the decimal place value style as follows:

aṭṭhārasa thānesuṁ sunnāniṁ do ṇavekka do ekko/
paṇa-ṇava-caukka-sattā saya-sattā ekka-tīya-sunnā//123//V
do aṭṭha suṇṇa-tia-ṇaha-tiya chakkā donni-paṇa caukkāni/
tiya ekka caukkāniṁ aṅka kamena palla romassa//124//V

Translation: Eighteen zeros in the last places, two, nine, one, two, one, five, nine, four, seven, seven, seven, one, three, zero, two and four. These are the digits of *palya*-hair in order. In the following circa 4th-5th century text, the symbols also occur, following the verse. The decimals with words appear as follows:

Word	Meaning	Symbol	Verse	Chapter
sada	hundred	100	125	1
lakkha	*lakh*	10000	156	
sagavisa	twenty-seven	27	168	
uṇavaṇṇa	forty-nine	49	168	
coddasa	fourteen	14	171	
dasa	ten	10	179	
vādāla	forty-two	42	182	
terasa	thirteen	13	186	
tettisa	thirty-three	33	191	
saya	hundred	100	191	
paṇṇattasi	seventy-five	75	198	
unadala	thirty-nine	39	198	
chappana	fifty-six	56	201	
ekkona vanna	forty-nine	49	214	
causīdī	eighty-four	84	229	
saṭṭhi	sixty	60	285	

[Note: The number 31980000 is expressed as twenty less one *lakh* eighty thousand, *tiṇhaṁ kodiṇaṁ*: Ch. 1, p. 123 (*TPT*(V))

"*egūnavīsa-lakkha-asidi-sahassa-tiṇhaṁ kodiṇaṁ*"

Thus, the only change in the decimal place for the denomination of *sada sahassa* has been to *lakh* in *TPT* and *DVL* from the *CKG*. In other denominations only declensions count.

The notation in decimals may be seen at peak in the denominations of time in years in Chapter IV of *TPT*(V). This is described in VV.294 et seg. They are as follows:

Denominations of Notation	Meaning	Remarks
dasa	ten	two *yugas* of four years each
sada	hundred	on multiplying by 10 from the preceding
sahassa	thousand	on multiplying by 10 from the preceding
dasa sahassa	ten thousand	on multiplying by 10 from the preceding
lakkha	*lakh*	on multiplying by 10 from the preceding
(*pūrvāṅga*) *puvvāṅga*	$(10)^5X84$	on multiplying the preceding by 84
(*pūrva*) *puvva*	$[(10)^5X84]^2$	on squaring the preceding or $7056(10)^{10}$
(*parvāṅga*) *pavvāṅga*	$[(10)^5X84]^2X84$	on multiplying the preceding by 84
(*parva*) *pavva*	$[(10)^5X84]^2X84X84X(10)^5$	on multiplying the preceding by $84(10)^5$
nayutāṅga	$[(10)^5X84]^2X84X84X(10)^5X84$	on multiplying the preceding by 84
nayuta	$(10)^{20}(84)^6$	on multiplying the preceding by $84(10)^5$
kumudāṅga	$(10)^{20}(84)^7$	on multiplying the preceding by 84
kumuda	$(10)^{25}(84)^8$	on multiplying the preceding by $84(10)^5$

The above process of multiplication with 84 and $84(10)^5$ goes on until ***mahalata*** is obtained. Then the process of multiplication by 84 *lakh* or $84(10)^5$ continually gives *Śrīkalpa*, *hastaprahelita* and ultimately *acalatma* whose value is $(10)^{90}(84)^{31}$ years. This is to be carried over to maximal numerate (***utkṛṣṭ saṁkhyāta***), of which the process of construction has been very elaborately described by R.C. Gupta, as maximal numerate plus one, or as the first maximal innumerate.[36]

It may be noted that for setting the measure of various types of sets in karma theory, there have been various types of constructed numbers as numerate, innumerate and infinite which have not been expressed to dazzling heights without any purpose. Hence it appears that the school needed a system of this decimal type in their theory.[37]

Words For Zero

First the words for zero are examined from *CKG, DVL* and *KPS, J DL*:

Word	Meaning	Value	Remarks
avahirijjanti[38]	making a set zero by reduction (exhaustion one by one)	zero	*DVL* 1, 2, 3. Concept of exhaustion one by one and 1-1 correspondence, p. 28. Book 3, vide p. 220, also
aṭṭha suṇṇa	for removal of zeros at the end	eight	*DVL* 1, 2, 4, p. 36
viṇāsaṇaṭṭhaṁ (tattha) avahiranti	making a set zero or or exhausted	zero	*CKG* 1, 2, 3, p. 27, vide also p. 230, inVV, 1, 2, 3, 4, 1, 1, 35
antarmuhūrta	tending towards muhurta in the limit of difference tending to zero	approximate	*DVL*, 1, 2, 6, p. 69
avaharraṭṭhā	seen symbols for division		*DVL*, p. 37,1, 2, 6, p. 87
saṭditthiṇa ditthā	values of denominations of time		
khavaga	destroyer or annihilator		*DVL*, p. 94, 12, 12, p. 94
anantā bhāgassa	infinitesimal	limit	*DVL*, p. 121, 1, 2, 15
anantim bhaga	part of infinites	to infinitesimal major part	
asaṁkha bhāga	innumerable part	limit to innumerably small part	*DVL*, p. 63, 1, 2, 16, p. 130
asamkhajjodi bhāga	innumerable part	limit to innumerably small part	*CKG*, 1, 2, 17
abhāva	absence	nil	*DVL*, 1, 2, 18, p. 160-1
suṇṇaṭṭhāṇa	zero place		*DVL*, 1, 2, 19, p. 181, 188
gayana	sky	zero	*DVL*, p. 71: 1, 2, 45
nabha	sky	zero	p.225 in place value

[Note: The verse depicting place value, from right to left.]

gayaṇṭṭha-naya-kaṣayā causatthi-miyanka-vasu-kharā-davvā/
chāyalā-vasa-ṇabhacda-poyattha-caṁdo ridū kamaso//71//

Translation: The number is given by taking in order eight zeros, *naya* (two), *kaṣayā* (sixteen), sixty-four, *mrgāṅka* (one), eight, *khara* (six), *dravya* (six) forty-six, eight, zero, *acala* (seven), *padārtha* (nine), *moon* (one) and *ṛtu* (six). This number is $6197084666816 4162(10)^8$. The verse projected is ancient and earlier to *DVL*, collected as such..

There is one more verse next, projected in *DVL*, showing the area of Jambu Island in the place value system, but left to right, in which the word *suṇṇa* (zero) occurs explicitly two times:

satta nava suṇṇa pañca-chattha nava caduekkaṁ ca pañca sunnaṁ ca/
jambūdivassedaṁ ganitaphalaṁ hodi nādavvā//72//

Translation: The area of the Jambu Island should be known (given by writing numbers as) seven, nine, zero, five, six, nine, four, one, five, zero (in square *yojanas*). The number is 7905694150.]

It may be noted that zero has not only been used in place value notation, but also defined set theoretically to give the concept of a set having no element or number, by exhausting a set with elements one by one. This is not only for instant-sets (*samaya rāśis*) but also for point-sets (*pradeśa rāśis*). For example, "*khetteṇa veindiya-tiindiya-cauvindiya tasseva pajjatta apajjattehi podaramavahirdi aṅgulassa asaṁkhejjadi bhāga vaggapoḍithāena*". Translation: Relating to region, two-sensed, three-sensed, four-sensed bios set-cardinals exhaust the universe square (*jagapratara*) as divided by the square of the innumerate part of linear finger-width (in terms of point-sets).

Word	**Meaning**	**Value**	**Remarks**
suṇṇa	zero	zero	*JDL*, 1, p. 33, p. 81, vol. 1

[Note: The verse carries zero here in a placevalue, while giving the number of letters in scriptural (*śruta*) knowledge (right to left)]

paṁcekka chakka ekka ya du-paṁca ṇava sunna sattatiya satta/
sunna du-caukka satacchacadu cadu aṭṭhekka suda vannā//33//

Translation: Five, one, six, one, two times five, nine, zero, seven, three, seven, zero, two times four, seven, six, four, four, eight and one (when placed in leftward sequence), give the letters of the scripture. The number is 18446744073709551615.]

suṇṇa	absence	zero	*JDL*, vol. 4, V. 22, p. 11
naṭṭha	abolish	zero	*JDL*, vol. 6, V. 22, p. 127

Note: The sentence is

kammapadesaānamuvasāmanaā-nikacaṇā-ṇidhatti karaṇāṇaṁ visohie vinasapaduppaya ṇaṭṭhaṁ

Translation: In order to abolish (annihilate) the karma particles, the operations of subsidence, preservation and reservation have been mentioned.

Now the *TPT* and *TLS* are sought for zero words:

Word	**Meaning**	**Value**	**Remarks**
suṇṇa	zero	0	*TPT*(V), 123, Ch. 1
ritta	vacant	0	V. 127. The concept of exhaustion of a set.
cauthāṇesuṁ suṇṇā.	zero in four places	0000	Ch. 3, V. 54, 86, 88
padaṭṭhāṇe suṇṇaṁ	zero in place of fost	0	Ch. 4, V. 53
natthi tudareṇu	zero in place of tudarenu (scale)	0	Ch. 4, V. 55
ambara	sky	0	Ch. 4, V. 59
ṇaudi uṇṇaṁgaṁ	on placing 90 zeros	$(10)^{90}$	Ch.4, V. 312

[Thousand is also given as ten hundred, V. 1199.]
[Note: *dasa ghana* means $(10)^3$ for 1000, V. 1170.]

ṇabha	sky	0 in unit's place	Ch. 4, V. 1173
gayaṇaṁbara	sky-sky	00 in two last places	Ch. 4, V. 1171
āyasa-ṇabha	sky-sky	00 in two last places	Ch. 4, V. 1175
ṇabha-ṇabha	sky-sky	00 in two last places	Ch. 4, V. 1176
ṇahba-aṁbara-gayaṇa	sky-sky-sky	000 in three last places	Ch. 4, V. 1176 from left to right

Note: The verse for example is as follows:

ṇabha-ṇabha-ti-cha-ekkekkaṁ aṅka-kame hoṁti savva vādigaṇā/
suttagaṇā ṇabha-aṁbara-gayanaṭṭha-caukka-aḍa doṇṇi//1176//

Translation: All debtors were (given by) zero, sky, sky, three, six, one and one (116300), in ordered digits. The total number of these seven classes is given by sky, sky, eight, four, eight and two (2848000) in sequence of digits.

gayaṇa ṇabha suṇṇa	sky-sky indirectness	00 for $(10)^2$ product placing in some of the boxes showing directness and indirectness between fordpounders and Cakravartis	Ch. 4, V. 1228 Ch. 4, VV. 1298-1302

Note: Here the presence of a Cakravarti in the period of a Tirthankara is shown by 1 and his absence by 0. Total boxes are 34.]

tha	zero	place second	Ch. 4, V. 2407, V. 2807

Note: The area of the Videha region as 2969349902/300
361 is shown in the following verse where *kha* occurs (4th to 5th century C.E):

du-kha-ṇava-cau-tiya-ṇava-chauṇava-duga joyaṇekka-pattīe/
bhāgā tiṇṇi sayā igi-chattiya-haridā videha khatta phalaṁ//

Translation: The area in square *yojanas* of Videha is given by the number (written from right to left) as two, sky (*kha*) nine, nine, four, three, nine, nine, six, two *yojanas* (square) in a line. (To this is added) three hundred divided by one, six, three (or 361).

khaṁ-ṇabha	sky-sky	00 or $(10)^2$	Ch. 4, V. 2682

Now we come tothe 10th-11th century work, *TLS*.

suṇṇa	zero	31 zeros or $(10)^{31}$	TLS, V. 21, p. 28

[Note: Here the number 19791209299968(10) is given in place value in words as *vitha, nidhi, naga, nava, ravi, nabha*, etc., from left to right.]

ṇabha-ṇabha	sky-sky	place value	*TLS*, V. 24, p. 31
na	alphabet na for zero	place value (left to right)	katapayadi system denoting zero, cf. *ASG*, p. 2

[Note: Here the number of hairs in a *palya* is given.]

gayaṇa	sky	zero	*TLS*, V. 309, p. 254
suṇṇa	zero	place value	*TLS*, V. 313, p. 261

Note: The number is in place value from right to left. It is one *krosa* more than 7905694150 *yojanas* (square).
Example:

chādalā suṇṇa sattaya uāvaṇṇaṁ hoṁti meru pahudīṇaṁ/
paṁcaṇṇaṁ paridhayaḥ kramanṇa aṁkakrameṇaiva//386//

ṇabha	sky	zero in place value	*TLS*, V. 770, p. 610 (right to left)
suṇṇa	zero	zero in place value	*JPS*, 11, 41, p. 189 (left to right)

JPS, 1.135, (right to left) *JPS*, 10.94, p. 183 (left to right)

[Note: In *TPS*, the number 6648658 780504 has been the combined area of the Koloda Ocean, but related as (V. 11.47)

chavaṭṭhiṁ aḍadālaṁ aṭṭhaṭṭhim sattasīdimasidiṁ ca/
paṇṇasaṁ ca caukkaṁ havadi ya kālodadhisaṇkhā//49//

Translation: The area of the Koloda Ocean (added with the area of Jambu Island, etc.) is sixty-six, forty-eight, sixty-eight, eighty-seven, eighty, fifty and four (square *yojanas*).

This is an example of stating the number in decimals, place value, pairing and from left to right. The pairing is also seen in V. 10. 96, p. 183 of the same *JPS*, where each digit is stated and the 10 in the end is stated as "*dasa*", and not as 10, from left to right. Thus, combinations of two, three, etc. digits were followed for the specific purpose of expressing numerical measures in different units of time and space in astronomy. Karma theory, however, required several types of units apart from the space and time units, even beyond the theory of matter requirements, as the bios phases were required to be related with particle phases of material (*pudgala*).

Notational and Decimal Key Words in the Earliest Works

The *Kasāyapāhuḍa sutta* of Gunadharācārya has been regarded as the earliest monumental work on the theory of karma (c. 1st century BC or later), about two hundred years earlier than the *Chakkhaṇḍāgama* of *Dharasenācārya's* disciples, Puspadanta and Bhūtabali (c. 2nd century AD). This text is in *Śaurseni* Prakrit and was produced from the third *Pāhuda*, "*Pejjadasa*" (Love and Malice) of the tenth *Vastu* of the fifth *Pūrva*, "*Jñana Pravāda*" of the fourth class *Pūrvagata* from the *Dṛṣṭivāda*, the twelfth *añga* of Jaina scripture. It describes a profound and unified system-theoretic technique founded on the set theoretic approach, gradually developed through symbolic operational details of relations between phases of karmic material particles in the form of *āsrava* (input), *sattva* (state), *bandha* (bond), *nirjarā* (output, etc.) over quantitative maps through matrices (vide *LDS*).[41] The tradition goes back to the South Indian Śravaṇabelgoḷa hill Candragiri, where ācārya Bhadrabāhu-I (c. 4th century BC)[42] along with his initiates, Maurya Emperor Candragupta (respective knower of 14 and 10 *Purvas*) spent twelve years in isolation, possibly for the purpose of recording karma theory through Brahmi and Sundari scripts (for mathematical expressions), some years earlier than Asoka's inscriptions began to appear in Brahmi.[43]

Word	**Meaning**	**Remarks**
dasama (*dasa*)	tenth	*KPS*, V. 1, p. 1, V. 86, p. 611
sadeasīde	one hundred and eighty	V. 2, p. 4
paṇṇārasa	fifteen	V. 2, p. 4
ekkārasa	eleven	V. 7, p. 8
aṭṭāvisaṁ	twenty-eight	V. 8, p. 9
sattarasa	seventeen	V. 27, p. 260
solasa	sixteen	V. 26, p. 261
bārasa	twelve	V. 26, p. 261
vīsa	twenty	V. 26, p. 261
ūnavīsa	nineteen	V. 28, p. 263
tirasa	thirteen	V. 35, p. 267
aṭṭharasa	eighteen	V. 45, p. 274
suṇṇaṭṭhāṇā	zero places (absence)	V. 49, p. 277 and the following
saṁkhejja sahassa	numerable thousand	V. 114, p. 611

[Note: The verse gives a rare type of place value in terms of a variable "numerable" (*samkhajja*) as follows

saṁkhajjā ca manussesu khīnamshā sahassasa niyamā/
sesāsa khinamshā gadisu niyamā asamkhajjaā//114//

Translation: Among the human (male) beings the annihilator serene visioned are numerable-thousand as per rule. In the remaining life-courses they are innumerable as per rule.

vassa sada sahassa	hundred thousand	V. 131, p. 760
saṁkhāe	numerable years	V. 131, p. 760
sadsasahassesu	innumerable	V. 131, p. 760
asamkhejjesu	hundred thousand	V. 131, p. 760

Note: The verse gives the life-time of bonds here:

vassa sada sahassāiṁ ṭṭhidisaṁkhāe du mohaṇiyaṁ tu/
bandhadi ca sadasahassesu asamkhejjesu sesaṇi//131//

Translation: In the inter-state of two instant-effect, (the bios) binds the charm-karma as present transition-proestablisher for life-time of innumerable-thousand years, and the remaining karmas are bound for life-times of innumerable-hundred -thousand years.

Note 2: The calculation is further carried on to infinity -- a proper infinite, as if of G. Cantor, detailed further in *LDS*.

bārasa ṇava cha tinni ya kittīo hoṁti adha va anaṁtāo/
ekkekkamhikasāu tige tige adhavā anaṁāo//V. 163, p. 806//

Translation: The tracks of fluorescent anger, etc. affections are twelve, nine, six and three or infinite. In every one of the affections there are three *kṛṣṭi* (tracts) respectively or there happen to be infinite tracts.

There is one more word *gaṇānadiyaṁteṇa* meaning beyond numeration or meaning innumerable (V. 224, p. 885).Up to this time of Gunadharācārya the verses were pronunciated and then their meanings were lectured (*KPS*, p.4). Professor A. Cakravarty has placed Kundakundācārya, author of *Parikarma*, a commentary on the first three sections of the *Śatkhandāgamā*, and it appears, according to the veneration he attained just after Guatma Gaṇdhara, as early as the 1st century B.C. E. By the time of ācārya Bhūtabali, we find in literature, *Matābandha*, the symbol for zero has been used for fitting up gaps in writing of detailed karma-bond theory (prior to Kundakundācārya) throughout the work. (vide *MBD* p. 41 et seq.). For example,

navari ṇiddā-pacalā oghaṁ/
thīṇagiddnio nucca 0 anamtanu 0 jahao aṁto 0/

Translation: Specifically, that of sleep, deep sleep is as the *ogha* (i.e., minimal, maximal inter-*muhurta*), the *somnambulation* (three), illusion, infinite pro-bonding (four), is minimal inter-muhurta (V. 115, p. 91. *MBD*).

This treatment shows that by this time zero had not been used as a decimal place value. The commentator, Kundakundācārya, author of *Parikarma* must have before him *CKG, MBD* and *KPS*. This sort of abbreviating the full words and sentences, in which seven volumes of *MBD* have been written, might have inspired him to start with a line and zero (a *daṇḍa* and *suṇṇa*) to go ahead of nine digits as ten. This is a conjectural possibility. However, although *Parikarma* has been quoted by saint scholar Virasenacarya, the text is not extant

Concluding Remarks

The above illustration gives only a part of various logical, mathematical and intuitionistic techniques adopted by the Digambara Jaina School for the expositoon of a scholastic system theory (Karma Siddhānta) which became a topic of discussion by L. Alsdorff. This school claims to be in possession of only small portions of the second and fifth *Purvas* of the *Drstivada* knowledge. Thus, it forces a rich source of material on the use of zero and its various applications apart from the decimal place value system. □

REFERENCES AND ENDNOTES

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2. *Ibid*, p. 26.

3. *Ibid*, p. 27

4. Needham, J. and Ling, W. *Science and Civilization in China*, Vol. III, Cambridge, 1959, p. 11. Needham finds also that from the time of King Asoka (3rd century) there was a steady development of the forms akin to the Hindu-Arabic numerals,the first three integers being written in just the same way as the Chinese, the first epigraphic evidence for zero in India being in the late 9th century, *ibid*, p. 10.

5. Cf. *ibid*, p.10.

6. C.f. *ibid*, p.12.

7. The *Tiloyapaṇṇaṭṭī* of Yativṛsabhācārya (C. 5th century AD) Vol. 1, 1943. Vol. 2, 1952, edited by H.L. Jain and A.N. Upadhya, Sholapur (abbr. *TPT*).

8. *The Chakkaṇḍāgama of Puṣpadanta and Bhūtabali Ācāryas* (abbr. *CKG*) (c. 3rd century C.E. with *Dhavalā* commentary of Virasenācārya (c. 9th century C.E.) (abbr. *DVL*), Vols. 1-16, Amarasti etc., 1939-1959.

9. *The Kaṣāyapāhuḍa of Gundadharācārya* (c. 1st century C.E.) (abbr. *KSP*) with *Jayadhavalā* commentary of Virasenācārya and Jinasenācārya, Vols. 1-14, edited by Pt. Kailash Chandra, Mathura, 1972.

10. *The Mahābandha* (abbr. *MBD*) of c. 2nd century C.E., Vols. 1-7, Kashi, 1947-1948.
11. He belongs to the end of the 10th century C.E.
(a) The Trilokasāra with commentary of Madhvacandra Traividya and Viśuddhamati Āryikā (abbr. *TLS)* Shri Mahavirji, 1976.
(b) *The Gommatasāra*, Vols. 1-4 (abbr. *GJK* and *GKK*) with Karṇaṭaka Vrtti of Keśavavarṇi (c. 13th century C.E.) and translation *Śamyakjñana Candrika* commentary of Ṭoḍaramalla in Hindi (c. 18th century C.E.), New Delhi, 1978-81. *ASG* will denote its *Artha Samdrstu.*
(c) The *Labhisāra* including the *Kṣapaṇasāra* along with *Ḍhūnḍhāri* commentary of Ṭoḍaramala (abbr. *LDS, KPN*), Agas, 1960, edited by Pt. Phool Candra Siddānta Shāstri. *ASL* will denote its Artha Saṁdṛṣti (symbolism for norms).
12. The first volume of the first project has been published by the S.S.M.K. Jain Trust, Nehru Park, Katni, (M.P.) in 1994.
13. Datta, B.B., Geometry in the Jaina Cosmogrophy, Springer-Verlag: *Quellen und Studion* B Nr. 14-Datta, 1-5, 1930, P. 1-10. Cf. also Bag, A.K, Symbols for Zero in *Mathematical Notation in India*, PCAHC, BANC, 1970, 247-254.
14. Singh, A.N. (with Datta, B.B.), *History of Hindu Mathematics*, Parts I and II, Bombay, 1962.
15.Kapadia, H.R., *Introduction to the Ganita Tilaka of Sripati*, Baroda Oriental Institute, V-XIX, 1937. Kapidia, H.R., *History of Nāgarī Numerals*, A.B.D.R.S., 19, 1978-89, 386-394.
16. Singh, A.N., Mathematics of Dhavalā-I, in the *Dhavalā,* Vol. 4, edited by H.L. Jain, et al., Amaravati, V-XXI, 1942.
17. Three examples are: (i) 79999998 is expressed as a number which has 7 at the beginning, 8 at the end, and 9 repeated 6 times in between (V. 51). (ii) 46666664 is expressed as sixty-four, six hundreds, sixty-six thousands sixty-six hundred thousands andfour *kotis* (V. 52). (iii) 22799498 is expressed as two kotis, twenty-seven, ninety-nine thousand four and ninety-eight (V. 53). Singh remarks that quotations like the above found in Jaina works point to the early use of place value notation in India and afford valuable evidence not obtainable from Hindu works. Cf. *Jaina Antiquary*, Vol. XV, Dec. 1949, no. ii 46-53.
18. Jain, L.C. Mathematics of the Tiloyapaṇṇaṭṭi (Hindi) in the *Jambūdvipaṇṇaṭṭi Samgraha*, Sholapur, 1958, 1-109. Cf. the *Tiloyapaṇṇaṭṭi* (*TPT*) of Yativṛsabhācārya, ed. Upadhye, A. Nand Jain, H.L., I (1943), II (1952), Sholapur. Cf. also the *Trilokasāra* of Nemicandra Siddhāntacakravarti (*TLS*) and its commentaries by Madhava Candra Traividya and Ṭoḍaramalla, Bombay (1910) and Shri Mahavirji (1976).
19. Artha Saṁdṛṣṭi chapters on the *Gommatsāra* and the *Labdhisāra* (*ASG* and *ASL*) of Nemicandra Siddhāntacakravarti in *SJC* (*Samyak Jñana Candrika*) commentary by Ṭoḍaramalla, Calcutta, c. 1919. Cf. also recent New Delhi and Agas Publications, the *Karnataka Vrtti* of the Gommatsara by Keśava Varṇi (c. 13th century C.E. in New Delhi publication (vol 1-4) contains more detailed symbolic material.

20. Cf. *Labdhisāra*, INSA project, Vol. I, Kotni, 1994, 68-70.
21. *Ibid*, p. 81.
22. Cf. *Dhavalā*, Vol. 10, P. 421 et seq. In the Bakhaṣāli manuscript the symbol stands for an unknown term. Datta, B.B., The Bakhaṣāli Mathematics, *BCMS*, XXI, 1929, 115-145.
23. Vide 9, p. 93.
24. *Ibid*, p. 70.
25. *Ibid*, p. 71.
26. Vide 8.
27. Cf. *Dhavalā*, Vol. 4, 1942, 1, 2, 14, 51, p. 98.
28. Vide 6
29. The verse is from *ibid*, Vol. 3, 1, 2, 45, 71, p. 255:

gayaṇaṭṭha kasāya causaṭṭhi aiyaṅka vasu kharā davvā/
chāyalā vasu nabhācala payattha cande ridu kamase//

30. Cf. *TPT* VV. 1176, 2402 for example: The numbers 116300 and 2848000 are given by

ṇabha ṇabha ti cha ekkekkaṁ aṅka kame honti savve vadi ga sattigāṇa
ṇabha ambara, gayaṇaṭṭha caukka aḍa donni//1176//

The area of Bharta kṣetra, 6021353 $\frac{294}{361}$ is given by

pañca titi akka duga ṇabha chakke aṅkakkasena joyaṇayā/
ekka cha ti harida cau ṇava duga bhāga bharahakhetta phalaṁ
(Kota edition) //2462//

31. Vide *ibid* 4.308, p. 178 (Sholapur edition).
32. Vide 7
33. Vide *ASG*, p. 20 and *LDS* project, Vol. 1, p. 90.
34. Vide *ibid* p. 90.
35. Cf. *TPT*, Ch. 1, 28, p. 113-132.
36. Gupta, R.C., The First Unenumerable Number in Jaina Mathematics, *Ganita Bharati* 14 (1-4), 1992, 11-24. This number is analysed by him to have been constructed through pits in units, tens and hundreds places. Cf. also *TPT*, p. 102-103 for an easier analysis by L.C. Jain.
37. Jain, L.C., *The Tao of Jaina Sciences*, Delhi, 1992, Vol. 1, p. 1-46. Vide also Kapida, H.R., *Introduction to Ganita Tilaka of Sripati*, Baroda, 1937, p. V-LXIX, for information on notational places in GSS, *Sthānāṅga*, *Jambūdvipa prajñapti*, *Sūrya prajñapti*, *Anuyogadvāra* and *Jivasamāsa*, etc., as also the *Jyotiṣa karaṇḍaka*, the *Tativartharājavārtika* and the *Svopajña bhāṣya* of the *Tattvārthadhigamasūtra*, wherein the names differ somewhere along with the process of constructions. But in all of them 10 has been used along with 84 for this purpose. It may also be noted that for finding out the value of *pi*, the number 10 has been used for finding the finer values of *pi* through difficult processes as may be seen in Gupta, R.C., "Circumference of the *Jambūdvipa* on Jaina Cosmography", *IJHS*, 10.1, 1975, 38-44. The originality of manipulation is evident therein. The number 84 is also connected with the generative places

called *yoni* or *jāti* which are 84 lac in number of various types of bios classified in several ways.

38. For example, the verse 1, 2, 34 in *CKB* is:

asaṁkhejjāhi asappiṇi-ussappiṇihi avahiranti kāleṇa//34//

Translation: Relative to time (the five-sensed subhuman bios set) is exhausted by innumerable-innumerate hypo-serpentine and hyper-serpentine (periods of time or instant-sets). The method of one to one correspondence is described here in *DVL*, p. 233, Book 3, 1, 2, 35.

39. *CKG*, V. 1, 2, 7, 9, p. 313, Book 3 (*DVL*).

40. This is the combinatorial work from 64 alphabets.

41. The details of these operations are given in detail in *GKK*, II, p. 675 and *LDS*. The Nikaita operation means that the *karma* perticles could neither be brought into rise trail (*udayavali*) nor could be brought with other configuration (*prakrti*) form nor could be uptracted or downtracted. The nidhatti operation means that the *karma* particles could neither be brought to the rise-trail(*udayavali*) nor could be transited operation in which the *karma* particles could not be brought into rise-trail.

42. Edited by H.L. Jain, Siddanta Shastri *Curni sutras of Yativṛṣabhācārya* (c. 5th century C.E.), Calcutta, 1995, (abbr. *KPS*).

43. Vide Jinasena,the Harivaṁsāpurāṇa, 9/24, and the *Mahāpurāna*, 16/103-105-109. The *Sundari* may be taken to be Hīnākṣarī and Brahmi as Ghaṇākṣarī, which were requires to be perfect for the learning of karma theory. Vide, Jain, L. C., Secret of Hinākṣarī and Ghanākṣarī (in the moon-cave of Girinagara), *Arhat Vacana*, 1 (21, 1988, 11-16).

Vide also Goyal, S.R. *Brahmi:An Invention of Early Mauryan Period in the region of Brahmi script*, ed. by S. P. Gupta and K.S. Ramacandra, Delhi, 1979.

For more details, Cf. Jain, L.C., *Arhat Vacana,* 2(2), 17-26, 1920, 2(3), 1-2, 1990 and 89, 92, ibid, 4 (1), 1992, 13-20, 5(3), 1993, 155-171.

44.Vide Jain, L.C., *Arhat Vacana,* 3(3), 1991, 33-91, ibid, 2(1), 1989, 7-12, ibid, 2(4), 1990, 82-92. These are on Kundakundācārya relevant to decimal place value. Even the use of 10 in finding the value of *pi* as the square root of 10 and its *vāsanā* (rationale) by Madhavacandra Traividya is in support of this conjecture. Cf. Gupta, R.C., *IJRS*, 21(2), 1986, 131-139.

For the use of place value in the subtraction of factors, vide Jain, L.C., *IJHS*, 24(3), 1989, 163-180.

45. *German scholars*, Vol. 1, Varanasi, 1973, 1-5

SATPRARŪPAṆĀ SŪTRA

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Continued from previous issue of Jinamañjari

In conformity with the *Jainägama* coming down from ancient times, verse - 2

> *Etto imesim coddasaṇham jīvasamāsāṇam maggaṇam maggaṇatthadāye*
> *Tattha imāṇi coddasa ceva ṭhaṇāni bhavanti* - 2
>
> On this basis of the *Āgamās*, the fourteen *mārgaṇasthānās* (stations of investigation) are worth knowing and search into the fourteen *jīva-samāsās* (spiritual stages) - 2.

Q. Which category of *mārgaṇasthāna*, physical or psychical is intended in this verse ?
A. According to the Jain tenets, the psychical type is intended.

Q. How one would learn about it ?
A. It is learnt from the term "*Etāni* (*imāni*)" in the verse, as explained and elaborated by Virasena in his *Dhavala* commentary. According to it, the directly perceived psychical *mārgaṇasthāna* is intended by the term *"imāni"* and the physical type is not intended as it is distant and variable with respect to location, time and nature. Thus, the non-omniscient can not have direct perception.

Q. What is the definition of *mārgaṇasthāna* ?
A. It is the method or topic of investigation wherein fourteen *mārgaṇasthānas* in association with eight *anuyogavārās* (disquisition doors) are applied to study existence, numeration etc. It is also explained in the *Gommatasāra Jīvakāṇḍa* (*GJ*), verse 141: "One should know that there are only fourteen *mārgaṇasthānas*; and *mārgaṇasthāna* is defined as topic of search through or under which the modes of the living beings,+ like the infernal modes etc are observed or indicated through scriptural knowledge."

+ The *Sat-khandagama* (*SK*) is the first ancient Digambara text to describe living beings through spiritual stages for which the author uses the term "*jīvasamāsa*" (abridgment or categorization of the living beings under spiritual stages). It seems that there are many ways of such categorization on different bases like senses, embodiments etc. Later, this term was designated as "*jīvastāna*" and "*guṇasthāna*" in order to avoid terminological confusion. The term "*guṇasthāna*" has later developed as quality- based.

Q. What are the fourteen *mārgaṇasthānas* ?
A. *Tam jaha* - 3, and they are enumerated in verse - 4:

Gai indiye kāye joge vede kasāe ṇāṇe sanjame dansaṇe lessāe bhavie sammatte saṇṇi āhārae cedi - 4

The living beings are studied under fourteen *mārgaṇasthānas*, which are: *gati* (destiny), *indriya* (senses), *kaya* (embodiments), *yoga* (activity), *veda* (libido), *kasāya* (passion), jnāna (knowledge), *samyama* (restraint), *darśana* (conation), *lēśya* (aureole), *bhavyatva* (liberatality), *samyaktva* (righteousness), *sanjnī* (instinctive) and *āhāraka* (intakership).

Q. Why there is in this verse the use of locative case in each of the terms like destiny ?
A. This is meant to denote that the *mārgaṇasthānas* like destiny etc. are the substratum for the living beings.

Q. Does the case of investigation stand scrutiny here as its four elements - investigator, object, method and means - being not mentioned, but are required for the study of anything in the world ?
A. Though the question is valid, the assumption of the absence of elements is not. The elements of investigation are found in the following contests: 1] The investigator is the faithful living being believing in the categories of the living and nonliving beings etc. 2] The object is the living beings and other entities. 3] The methodology is the investigation of destiny etc. which form the substratum for the objects. 4] The means is the teacher.

Q. Why this verse has described only the investigations and not the other three elements ?
A. Since the investigations are invariably related with the other three elements, their inclusion is found within the description of investigation.

Q. What is the definition of *gati* ?

A. It is the specific mode of the existence of the living beings or souls. Alternatively, it can be said that it is the movement from one state of birth to another state of birth. The *Prākṛta Pancasangraha* (*PP*) verse 1.58 also defines it this way: "The specific activity or movement of the living being due to the fruition of the physical-making karma is *gati*. Alternatively, *gati* is the instrumental cause for moving living beings in four states of infernal etc."

Q. What is the definition of *indriya* ?

A. It is defined as that agent which is engaged in its own object and which does not engage itself in the objects of other *indriyas*. In other words, *indriya* is the master of its own objects. *GJ* verse 164 also explains it this way: "The *indriyas* are like Ahamindra (I an lord), who has no classes of lords and servants, but feels as the lord of deities. *Indriyas* are capable of causing the knowledge about their own objects, independent of other *indriyas*."

Q. What is the definition of *kāya* ?

A. It is defined as *pudgala* in the form of gross body etc. (there are five types of bodies) accumulated due to the action-based propensity of the soul. The *PP*, verse 1.75 explains it this way: "Know, it is the *pudgala* in the form of gross body etc. accumulated due to the action-based propensity of the soul. Jain canons speak of six kinds of them, in the form of earth, water, air, fire, plant and mobile, and they form two categories namely, non-mobile and mobile *kāyas*. The first five kinds belong to the first category and the sixth kind belongs to the latter category."

Q. What is the definition of *yoga* ?

A. It is defined as the generation of potency instrumental in receiving karma due to three-fold propensities of the living being. Alternatively, it is the process of expansion and contraction of the space-points of soul (due to its propensities). It is also said in the *PP*, verse 1.55: "Jinas have spoken of it as the effort in terms of potency (or energy) instrumental in receiving karma by living being associated with mental, vocal and physical actions on its own. Alternatively, it is defined as motions or vibrations of the space points of the soul."

Q. What is the definition of *veda* (libido) ?

A. It is the rising of infatuation in the form of normal copulation or sex activity. It is also said in the *GJ*, verse 272 in this way: "The living being or soul performs many follies due to the normal or premature fruition of karma due to *veda*. Accordingly, experiences or the feelings due to the fruition of this activity is termed as *veda*."

Q. What is the definition of *kaṣāya* (passion) ?
A. It is the agency which is instrumental in ploughing, tilling and seeding the field of karma which produces various types of grains of pleasure and displeasure. It is also said in the *GJ*, verse 282 in this way: "*Kaṣāya* is defined as the agency which ploughs the field of karma producing many types of grains of sorrow and happiness and whose boundary is very large and far away (in terms of birth cycles)."

Q. What is the definition of *jnāna* (knowledge) ?
A. It is defined as specific energy or capacity which enlightens about true realities and through which living being learns about the reals and realities along with their attributes and modes. The *GJ*, verse 299 defines it in this way: "It is an agency through which the living being perceives or learns all about all the three-time existing reals, their attributes and modes directly or indirectly."

Q. What is the definition of *sanyama* (restraint) ?
A. It is defined as the process of accepting and observing vows and carefulness, control of passions, renunciation of all types of violence and winning over the senses or sense objects. Verse 465 in the *GJ* also defines it as: "It is the process of accepting five major vows - *ahimsa*, *satya*, nonstealing and *aparigraha*; observance of five carefulness in walking, talking, food-intakes, picking and placing and excretions; control of four passions - anger, pride, deceit and greed; renunciation of mental, vocal and physical weapons of *himsa*; winning over of five senses of touch, taste, smell, sight and hearing."

Q. What is the definition of *darśana* (conation) ?
A. It is defined as internally oriented vision or enlightenment of consciousness. In contrast, the knowledge is defined as the externally oriented enlightenment of consciousness.

Q. The term *cit* (consciousness) is defined as the experiencing of the nature of the self-soul in accordance with the karmic destruction-cum-subsidence. In contrast, the term *prakāśa* (enlightenment) is defined as

the knowledge of external objects different from the self-soul. Moreover, the knowledge is defined as the medium through which the living being learns the nature of the self and the external objects. Is then the difference between *darśana* in terms of internal enlightenment and knowledge in terms of external enlightenment is unproven ?

A. There is difference between *jnāna* (knowledge) and (*darśana*) conation, and no enlightenment of objects occurs through knowledge like "is this an earthen pot" or "this is a fabric" etc.

Q. It should then be admitted on this basis that *darśana* (conation) perceives the inner and external generality while *jnāna* perceives the inner and external particularity.

A. The particularity without generality is not capable of *artha-kriyakāri* (casual efficiency). Further, an entity which is incapable of casual efficiency is a non-entity. Hence, the knowledge perceiving a non-entity cannot be called valid. Similarly, the conation perceiving only the generality can also be not valid. All this means that the particular devoid of general and the general devoid of particular is a non-entity. Knowledge perceiving particular devoid of only general, and conation perceiving general devoid of only particular can be admitted as a valid cognition ? So knowledge is that which perceives general-cum-particularized eternal; and conation is that which perceives general-cum-particularized inner self.

Q. If one accepts the above definition of *darśana* and *jnāna*, will it not be contradictory with the scriptural definition of *darśana*, which perceives the entities in general (vide the *GJ* verse 543) ?

A. The term general in this definition has been made to indicate the self as it is the common substratum for all the external entities. To confirm this point, the *GJ* verse 543 has given a term with an adjective meaning in terms of "not perceiving the shape, etc (i.e. details) of entities." It means that the perception of generality irrespective of particularity is called *darśana*. It is also said in the *GJ* verse 543 that the Jaina scriptures have defined *darśana* as that which grasps or perceives the general-cum-particularized external entities in general without individual differentiation points or details.

Q. What is the definition of *leśyā* (aureole) ?

A. It is the mental, vocal, or body propensity painted (or associated) with passions. It means that it is neither propensity passions nor action-based propensity. However, it should not be taken to mean that there will

be no *leśyā* in the detached ones in the eleventh or higher spiritual stages, as the *leśyā* is activity-prominent rather than passion-prominent. *Leśyā* is the adjective of activity. It is also said in the *GJ* verse 489: "The saints knowing about the nature of *leśyā* have said that it is defined as that medium through which the living being besmears himself with sins and sacred."

Q. What is the definition of ***bhavya*** (liberatable) ?
A. It is the individual who is capable of attaining liberation from this world. The contrast is the individual who is non-liberatble being incapable of attaining liberation. The *GJ* verse 558 also states that there are two kinds of liberatables - those who have already attained liberation and those who have the capacity to attain.

Q. What is the definition of *samyaktva* (righteousness) ?
A. From the pure or absolute standpoint, it is defined as the faith in the reals and realities with reference to: pacification, desire for salvation, compassion and belief in scripture and rebirth. It is also said in the *GJ* verse 561: "One who believes in six realities, five existents and nine reals as sermonized by the Enlightened and described in the scripture. Or, it may be the belief in the above through the methods of valid cognition, standpoints and positing."

Q. What is the definition of *sanjnī* (instinctive) ?
A. The instinct or mind is defined as that which knows or feels well. The *sanjnī* being is that who has the mind (physical or psychical). A non-instinctive does not have any type of mind. Hence the instinctive living being receives education, performs actions and receives or delivers instructions and speaks with the help of mind.

Q. What is the definition of *ahāraka* (intaker) ?
A. The living being who acquires mattergic mass suitable for the growth of bodies like gross bodies, etc. is a *ahāraka*. It is also said in the *GJ*, verse 665: "That living being who receives *spardhaka pudgala* (of mattergic supervarifirm) karma suitable for one out of the three bodies of the nature of gross protean and projection and for the growth of speech and mind."

Q. What is the definition of *anāhāraka* (non-intaker) ?
A. It is he living being who does not receive mattergic mass for the formation of growth of gross body etc. It is also said in the *GJ* verse 666

in this way: "The living beings of [i] all the destinies under transmigration, [ii] potentially omniscient, [iii] static omniscient and [iv] liberated ones, as a rule, are *anāhārakas* while remaining ones are *ahārakas*."

To describe *anuyogadvaāra* for the study of spiritual stages, objects of investigation is found in the following verse:

> *Edesim ceva coddasanham jīvasamāsāṇam parūvaṇatthādaye tattha imāni aṭṭha aniyogaddārāṇi ṇāyavvāṇi bhavanti* - 5
> Following eight *anuyogadvaāras* are worth knowing to enunciate or study of fourteen spiritual stages (as indicated in v.2)

> *Tam jaha.* 6
> Eight *anuyogadvaāras* are described in the follwing v.7

> *Śantaparūvaṇa,davvapamāṇanugamao, khettānugamo, phosanāṇugamo, kālāṇugamo, antarāṇugamāmo, bhāvāṇugāmŏ, appābahugāṇūṇgamo cedi* - 7*
> Eight *anuyogadvāras* are: [I] enunciation of existence [ii] confomatory explanation on numeration of realities (and reals) [iii] conformatory explanation on location [iv] conformatory of explanation on contact [v] conformatory explanation on time [vi] conformatory explanation on interval [vii] conformatory explanation on current mode or disposition and [viii] conformatory explanation on relative numeration.

It is also said in the scriptural verse (no reference given) that [I] the description of the existence of reals and realities is called enunciation of existence [ii] the description of the numeration of the categories of existents which have been known to be in existence is explanation on numeration (reals and realities [iii] the description about the current location of the reals and realities is explanation on location [iv] the description of the past and present contact (of reals and realities with each other) is explanation on contact [v] the description of minimum and maximum duration of the existence of realities and reals is explanation on interval [vi] the description of time interval or zero interval about the realities and reals is explanation on interval [vii] the description of changes, transformations, dispositions and current state, etc of the entities in the world is explanation on mode [viii] the term relative numeration referring to entities in the world is self-explanatory.

Verse - 8 explains the first *anuyogadvāra - sat-prarūpaṇao*:

Santa-parūvaṇadāya duviho niddeso-oghena asesena - 8
There are two kinds - [I] general and [ii] particular - 8

From verse - 5 above, the meaning of v.8 should be taken as: There are two types of descriptions - general and particular, describing general enunciation (of realities and reals) in case of the former and particularities in the latter.

* K.C. Shastri seems to have taken the terms *anugama* and *prarūpaṇa* as synonymous. That is why he has used the term *prarūpaṇa* while giving the sense of the quoted verses. However, it should be kept in mind that the term *anugama* clearly means explanation in conformity with the scriptures while the term *prarūpaṇa* means the excellent examination or explanation. This excellence can arise only when the descriptions are conforming to the scriptures. Thus the idea of conformity with scriptures is automatic in this term. While defining *anuyogadvāra*, the word conformity should be taken in each case, and they are the methods of studies of any entity.

In general, *anuyogadvāras* could be applied to the studies on all realities and reals, which the early Jaina texts have employed them for the studies on the living beings. Later Jaina scholar Akalanka employs them to the non-living beings also.

Q. How to define *jīvasamāsas* (spiritual stages) ?
A. This term is the substratum in which the living beings are observed to be existing during the *bhāva* (modal) development.**

** They are described in the *Tattvarthasutra* and other. According to some scholars, there are six types where they add mixed or conjunctional mode. However, five kinds are popular in the tradition. Though these modes have karmic and non-karmic origins in the scriptures, currently they form important part of psychology in explaining, controlling and improving human behaviour or mental states, such as repression, regression, denial, projection, sublimation, displacement, rationalization, redirection, compensation and dissolution. According to the religious interpretation, better or purer the *bhāva* better will be the religiosity, happiness and spirituality, and *jīvasamāsas* are specific steps.

Q. Where do these living beings exist or reside for their purpose ?
A. They reside in their qualities or attributes which are developed gradually through *jīvasamāsas*.

Q. What are the qualities of the living beings ?
A. The term quality here stands for mental dispositions due to karmic or non-karmic causes, and they are of five kinds: [I] *audayika* (fruitional) [ii] *aupasamika* (caused by karmic subsidence) [iii] *kasayika* (caused by karmic destruction) [iv] *kasayika-aupasamika* (part destruction and part subsidence of karma) and [v] *parināmika* (inherent).
The attributed living being is also designated as the attribute or *guṇa* (quality) because of the concomitant association. It is also said in the *GJ* verse 8: living beings are named as "quality" or "attribute" by the omniscient.

Verse - 9 describes spiritual stages in general and the first stage in particular:

Ogheṇna atthi micchāiṭṭhi - 9***
In general, there are wrong-faithed living beings -9

***In Prakrit, the term *atthi* may have meanings in both singular and plural numbers - is and are, which continue up to verse 23. The term *micchāitthi* in here and in the followings up to 23 will have expressions in plural number, as well expressed in terms of abstract noun or attribute (wrong faith) or adjective (wrong-faithed). In this context, "attributed" is implied as the living being.

Q. Who is a *mithyādraṣti* (wrong-faithed) one ?
A. Those living beings who are have predilection towards falsity or irrationality are called wrong-faithed. It is explained also in the *GJ* verse 17: "The living being has reverse or false predilection due to the experience of wrongfulness arising because of the fruition of the wrong - faith. He does not beieve the true religion in the same way as the man with bilious fever does not taste even the sweet juice."

Second *guṇasthana* is described in verse - 10:

Sāsaṇa-sammāitthī - 10
In general, there are the sasādanis (who linger on right-faith) -10.

The terms *oghena* (in general) and *atthi* (are) are followed here from verse - 9 to get the correct meaning, and are followed up to verse .

Q. What is the definition of *sasādani* ?
A. The opposition to, or offending of right faith with denial or *āsādana* (lingering) is called *sasādani*, whose right faith has been

vanished due to the fruition of the infinite-bonding passion, and has not developed firm *bhāva* about wrongness.

Q. As there are three types of faiths - [I] right [ii] wrong [iii] *ubhayarupa* (mixed). *Sasādani* (lingering right-faithed) is neither *mithyadraṣt* as he does not accumalte karma, nor is he *samyakdṛṣti* (right-faithed); he is not *ubhayarupi* (mixed) type. Since there is no fourth kind of right-cum-wrongness, why there should be a category of *sasādani* in spiritual stage ?

A. In *āsādana* there is false predilection, which has two kinds: one arising from infinite-bonding *kasāya* and the other from karma of wrong faith. The *āsādana* having infinite-bonding *kasāya* therefore is wrong-faithed. But there being no wrong predilection arising out of wrong-faith, *āsādana* is right-faithed rather than wrong-faithed.

Q. How could it be right-faithed when there is wrong predilection arising from wrong-faith ?

A. It is because of the fact that the living being was right-faithed earlier. This is also has been said in the *GJ* verse 20: *Āsādana* living being in his second spiritual stager approaches wrong faith, falling from right faith. It means that his right-faith is vaning but has yet to acquire wrong-faith completely.

The next verse describes the third spiritual stage of *samyag-mithyādraṣti* (right-cum-wrong faith):

> *Sammā-micxchāitthi* - 11
> In general, there are the right-cum-wrong-faithed living beings .

Q. What is the definition of *samyag-mithyādraṣti* ?

A. He is called so for he is right-cum-wrong-faithed as having both types of faiths: right-cum-wrong.

Q. As it is not possible to have both types of faiths at a time in a living being, does third spiritual stage stands scrutiny?

A. When it is possible to have right and wrong faiths in a living being in succession, it could also be possible to have both types of faith in him at the same time. We find such right-cum-wrong-faithed who hold that the enlightened ones are also venerable deities, without resigning from the worship of their earlier accepted deities.

Q. Which is that *bhāva*, (volition) of the five stated earlier, in this third stage ?
A. There is this *kṣayopaṣamik bhāva* (destruction-cum-subsidential).

Q. What is the relationship of this *kṣayopaṣamik bhāva* with living being crossing from *mithyātva* (wrong) to *samyag-mithyātva* (right-cum-wrong) *guṇasthana* (spiritual stage) ?
A. *Samyag-mithyātva* is acquired due to [i] *udayabhāvi kaṣāya* (non-fruitional destruction), *aupamāsika* (subsidence) all-destroying *spārdhaka**** (large group of karmic atoms - supervariform) of *mithyātva-karma* (wrong-karma); and of the karma of right-cum-wrong-faith. Hence the relation.

*** The karmas are not ultimate atoms but are larger groups called *varganas* (variorums) and still larger *varganas* are called *spardhakas* (supervariforms). And the karma may be in the form of either of these two depending upon the intensity with which they are earned.

Q. As there is karmic reap from *samyag-mithyātva*, why the *audayika bhāva* (fruitional volition) has been stated in this stage ?
A. Just as there occurs a total destruction of *samvyaktva* (right faith) on account of *mithyātva* (of wrong faith), there is no total destruction of right faith on account of *samyag-mithyātva*. Therefore, this third *guṇasthana* is called *kṣayopaṣamika bhāva* (destruction-cum-subsidential volition) instead of calling it *audayika*.

Q. When the reap of *samyag-mithyātva* (right-cum-wrong faith) does not totally destroy the right faith, why it has been referred as all-destroying (in terms of supervariform) ?
A. Karma generated by *samyag-mithyātva* only limits the totality of right faith. It is with this point of view that right-cum-wrong faith has been called all-destroying. It is also said in the *GJ* verse 22: "Just as a mixture of curd and jaggery cannot be separated into constituents, mixed *bhāvas* in the form of right-cum-wrong faith can not be separated. Hence, mixed volition is the third stage of mixed faiths."

Following verse describes the fourth *guṇasthāna* of *asanyata* (non-restrained) *samyagdraṣti* (right faith):

Asanjada-sammāiṭṭhi - 12
In general, there are *asanyata* (non-restrained) *samyagdraṣti* (right-faithed) living beings.

Q. What is the definition of *asanyata asanyata samyagdraṣti* ?
A. A *samyagdraṣti* is one who has a right or true belief, vision or predilection; is and is of three kinds [i] *kṣayak* (destructional right-faithed) who has completely destroyed seven karmic species, and never acquires wrong faith nor doubts scriptures. [ii] *vedaka* (destruction-cum-subsidential right-faithed) [iii] *aupaṣamika* (subsidential right-faithed) who may become wrong-faithed, lingering right-faithed or right-cum-wrong-faithed.

Q. Of the five *bhāvas,* which one is there in the fourth *guṇasthana* ?
A. The right faith arising out of the destruction of the seven karmic species is *kṣayak* (destructional right-faith). The right faith arising out of subsidence of the same seven species is *aupaṣamika* (subsidential right-faith). The right-faith experiencer is *vedaka-samyaktva-kṣayo aupaṣamika* (destruction-cum-subsidential), which arises due to the fruition of the species of righteousness (including right faith) leading to experience of partial destruction of right faith.

Q. Why is adjective *asanyata* (non-restrained) for the right-faithed?
A. This adjective is terminal ending, and indicates that the first three *guṇasthānas* are also non-restrained along with the fourth stage.

Q. Why this adjective *asanyata* does not indicate its character in the fifth *guṇasthāna* ?
A. It does not indicate this point because higher stages have either *guṇasthānas sanyama-asanyata* (restraint-cum-non-restraint) or *sanyama* (restraint) only. It is also said in the *Gj* verses 27 and 29 in this way: "Normally the right faithed living being does believe in the sermons of the Jinas, but he may also believe the wrong or contrary instructions from the teachers out of ignorance or without knowing the sense. The right-faithed *asanyata* living being believes in the sermons of the Jinas."

Next verse describes *guṇasthāna* of *deṣaviraṭi* (partial restraint or abstinence):

Sanjadā-sanjadā - 13
In general, there are *sanyama-asanyata* (restrained-cum-nonrestrained living beings).

Q. What is the definition of *sanyama-asanyata* ?
A. Those living beings who are *asanyata* (non-restrained) despite being *sanyama* (restrained) also.

Q. The *sanyama* living being cannot be *asanyata*, and vice versa as these terms are mutually contradictory. Hence, is there the possibility of the existence of this fifth spiritual stage?
A. There is no contrariety in agreeing to the existence of restraint and nonrestraint in a single living being. Because, the factors for their origin are different. The restraint is caused due to the abstinence of violence of mobile beings and the nonrestraint is caused due to non-abstinence of violence of the non-mobile beings. It is also stated in the *GJ* verse 31: "The living being is said to be rstrained-cum-nonrestrained who is abstained from violence of mobile beings and is non-abstained from violence of the non-mobile beings at the same time despite his faith in the Jinas and their sermons."

Verse14 describes the sixth stage of *pramāda* (nonvigilant) *sanyama* (restraint):

> *Pamatta-sanjadā* - 14
> In general, there are *pramāda sanyama* (non-vigilantly restrained) living beings.

Q. What is the definition of the term *pramāda sanyama* ?
A. It is intoxication by prosperity (*mada-mad*; pra-posperity). In contrast, restrained living beings violence in a proper.

Q. If the sixth stagers are *pramādis* (non-vigilant) they can not be restrained as they can not know the true nature of their self. If they are restrained, they can not be non-vigilant as the restraint can be there only when the non-vigilance goes away. Hence, how could there be a spiritual stage like the sixth one ?
A. Five sins - *himsa*, *asatya*, stealing, nonchastity and *parigraha* - are not annihilated by *pramāda* (non-vigilance), which, however, may cause some flaws or mutilations in the *sanyama* (restraint).

Q. How it could be ascertained that flawed *sanyama pramāda* (restrained-nonvigilant) is intended and not the *kṣiṇa sanyama pramāda* (restraint-destroying nonvigilance) in this sixth spiritual stage ?

A. There is no possibility of *sanyama* (restraint) in the presence of *kṣiṇa sanyama pramāda* in the sixth stage and therefore, it ascertains that it is only the flawed *pramāda* (novigilance) is intended in here.

Q. Out of the five modes, which *bhāva* (mode) is in this stage?
A. There is *kṣayopaupasamika bhāva* (destruction-cum-subsidential) in this stage as *sanyama* (restraint) is caused due to [i] *udayabhavi* (non-fruitional) karmic drippings of the currently existing *kṣiṇa kaṣāya* [ii] subsidence of existing karmic future fruition [iii] result of the gathering passion.

Q. When *sanyama* (restraint) is there due to gathering passion, why it is not stated that there is *audayika bhāva* (fruitional mode or volition) in this stage ?
A. Because, *sanyama* is not caused only as a result of gleaning passion.

Q. What is then the function of the fruition of the karma of gleaning passion here ?
A. It mutilates *sanyama* only. It is also said in the *GJ* verse 33: "The living being is called *pramāda sanyama* (nonvigilantly restrained) who manifests and non-manifests physically and psychically); conduct oneself and adheres to *mahāvratās*."

The next verse describes the seventh spiritual stage of guarded *sanyama* (restraint) among all the types of *kṣayopaupasamika* (destruction-cum-subsidential restraints):

> *Appamatta-sanjadā* -15
> In general, there are apramat-sanyat (vigilantly restraint) living beings.

Q. Who are *apramat-sanyats* ?
A. They are those whose restraint is devoid of *pramāda (*nonvigilance). It is also said in the *GJ* verse 46: "The living being -- who has destroyed all *pramāda*; conducts oneself with vows and attributes; and absorbs in meditation (of the third and fourth type), but holds steady without going up or down the spiritual ladder -- is called *apramat-sanyat.*"

The next verse describes the eight spiritual stage:

Apuvvakaraṇa-paviṭṭha-suddhi-sanjadesu atthi uvasamā khavā - 16
In general, there are karmic subsider and destroyer living beings purified through *apurvakāraṇa-sayana* (unprecedented volitions).

Q. What is the definition of *apūrvakaraṇa-sayanat* ?
A. The terms ***karaṇa*** here means mental disposition or ***bhāva*** and *apūrva* means as nonexistant or unprecedent. Thus the word *apūrvakaraṇa* indicates that there are innumerable types of mental dispositions with respect to many living beings varying gradually in each *samaya*, from the very beginning. However, mental dispositions of the living beings of given time are unusually different from that of the mental dispositions of the living beings of any intended time. Thus *apūrvakaraṇa bhāva* are those which did not exist at previous times and have unusual character at each time. The living beings with such *apūrvakaraṇa bhāva* are called *apūrvakaraṇa-sayanat* (unprecedented volitionally restrained). They could be *aupaṣamika* (karmic subsiders) and *kṣapak* (karmic destroyers) because of his tendency towards destruction or subsidence of karma.

Q. In the eigth stage neither there is *kṣaya* (destructional) nor *upaṣam* (subsidence) of karma. Hence how the living beings at this stage could be called *aupaṣamika* and *kṣapak* ones ?
A. As a rule, the living being at eighth stage does destroy or subside the conduct-deluding karma in future. Hence, the living being at this stage has been formally or figuratively called *kṣapak* (derstroyer) or *upaṣamak* (invader) of karma.

Q. What is the *bhāva* at this stage out of the five kinds ?
A. There is the destructional volition in *kṣapak* being and there is the *aupaṣamika* (subsidential) volition in *upaṣamak* being. It is said also in the *GJ* verses 51, 52 and 54: "The Jinas have said that there is never any similarity in volitions of the living beings existing at different times in the stage of *apūrvakaraṇa* (unpreccedented) *bhāva*. However, there is both similarity and dissimilarity in *bhāvas* of the living beings existing at the same time. In this spiritual stage therefore the living beings existing at different times have *apūrvakaraṇa bhāva*. The living beings with such volitions are always ready for *upaṣam* (subsidence) or *kṣaya* (destructional) sub-species of the *mohanīya* (deluding) karma."

The next verse describes the last stage of the spiritual stages involving coarse *kaṣaya* (passions):

Aṇiyaṭṭhi-bādara-sāmparāiya-pavittha-suddhi-sanjadesu atthi uvasama khavā -17*
In general, there are *upaṣamak* (invader) and *kṣapak* (subsider) of karma living beings who realize greater purity.

* The meaning of the term *sāmparaya* is passion and the term *bādara* means gross that lead to the conjoined word gross-passions. The living beings associated with gross-passions could be subsiders or destroyers of karma, at this stage some subsiding some species of the deluding karma and destroying some other species in future. This spiritual stage is of karma subsidential and destructional.

Q. Who is called the *aniyatthi*?
A. The term *nivṛtti* in here means dissimilarity and the term *aniyatth* or *anivṛtti* will mean the opposite of dissimilarity. This means that there exist living beings with *anivṛtti kāraṇa* (similar volitions) and *nivṛtti* (dissimilar volitions) existing at the same time and there are only of the living beings existing at different times. Such a living being is called *aniyatthi*.

Q. Why there are no separate spiritual stages for *upaṣamak* (subsidence) or *kṣamak* (destructional) types ?
A. This is because there is similarity with respect to volitions in both the cases. This is explained in the *GJ* verses 56-57: "The living beings existing at the same time are mutually different with respect to their shapes of body etc. and there is no difference in their volitions. They are called *anivṛtti kāraṇa jiva* (similar volitioned living beings) They always have similar volitions every instant of time involving infinite-times volitional purity. These living beings are destroyers of the forest of karma through the fiery flames of very pure meditation."

Next verse describes the last spiritual stage of the *kuṣeela* (conduct-stained) monks:
Suhuma-sāmparāiya-paviṭṭha-suddhi-sanjadesu atthi uvasamā khavā - 18
In general, there are karmic subsider and destroyer living beings purified through volitions involving subtle passions.

Q. What is *suhuma-sāmparāiya* ?
A. The subtle passions are called *suhuma-sāmparāiya* and there are both *upaṣamak* (subsidence) or *kṣamak* (destructional) types among them. In this spiritual stage, the living being which destroys, will destroy and has destroyed many of the karmic species has *upaṣamak bhāva*

(destructional volition). Similarly, it, which also subsides, will subside and has subsided many of the karmic species in this stage has *aupaṣamika bhāva* (subsidential volition). The *kṣayika bhāva* living being possess destructional volition with respect to *samyag-darśana*. In contrast, the living being on subsidential ladder has both *upaṣamaka* and *upaṣamak* volitions, as he can ascend the subsidential ladder through both the categories of righteousness.

Because of both adjective terms-unprecedented and similar (in v.16 and 17 above), this spiritual stage has altogether different types of volitions in comparison to the earlier stage. It is said also in the *GJ*, verse 59: "The subtle-passioned living beings with *sūkshma-samparāya* (subtle nominal desire), which is the first pure contemplation, have *apūrva-kāraṇa* (unpreccedent) *spardhaka* (large group of karmic atoms - supervariform)."

The next verse describes the last spiritual stage of *upaṣamaśreṇi* (subsidential ladder):

> (*Uvasanta-kasāya-vīyarāya-chadumatthā* - 19.
> In general, there are living beings - *upaṣānta-kaṣāya-vitarāgi*
> (detached non-omniscient with subsided passions).

Q. What is definition *upaṣānta-kaṣāya-vitarāgi* ?
A. One whose passions have gone subsided is called *upaṣānta-kaṣāyi* and his detachment destroyed, he is called *vitarāgi*. The *jnānāvarṇiyia* (knowledge-obscuring) and *darṣanāvarṇiya* (conation-obscuring) karmas are called *chadma* (disguise), and the person under these two obscurations is called *chadmasta* (non-omniscience). With the adjective word 'detached,' *vitarāgi*. *chadmasta* is excluded up to the tenth *guṇastāna*. There is the adjective of 'subsided passions' which excludes the twelfth higher stage individuals.** It is also said in the *GJ*, verse 61: "The spiritual stage of *upaṣānta-kaṣāya* (subsided passions) is the purified *bhāva* form arising due to the subsidence of *mohaniya* (deluding) karma in totality, like the muddy water is purified by *kataka* fruit (nut-plant), or the clean water of the pond in the autumn."

** The duration of this stage is just 48 minutes. Afterwards, the living being at this stage falls down to lower stages due to the completion of life span or completion of the duration of this stage.

The next verse describes *nirgrantha* (passionless) spiritual stage:

Khīna-kasāya-vīyarāya-chadumatthā - 20.
In general, there are *kṣiṇa-kaṣāya-vitarāga-chadmastha jivās* (detached non-omniscience with destroyed passions).

Q. What is the definition of *kṣiṇa-kaṣāya- vitarāgi-chadmastha jivās* ?
A. One who is *kṣiṇa-kaṣāyi* (passions destroyed) and the *vitarāgi* (detached non-omniscient) is called *kṣiṇa-kaṣāya- vitarāgi-chadmastha* despite being detached due to destroyed passions.

Q. A *kṣiṇa-kaṣāyi,* no doubt is a *vitarāgi.* Then how the word vitarāgi should be understood ?
A. The word *vitarāgi* has been associated with this stage to indicate the fact that this stage refers to the psychical destruction of passions rather than namal - representational or substantive destruction of passions. The Jinas have said that the passionless living being of the psyche stage has become pure like the water kept in a quartz vessel.

Q. Out of five *bhāvās,* what kind is found in this stage ?
A. Prior to attining this stage, there is the total destruction of *mohaniya* karma. Hence, this stage has *kṣiṇa-bhāva* (destructional volition). It has been also said in the *GJ,* verse 62, which has been explained in the precceding explanation.

The next verse describes the thirteenth spiritual stage of *sajogi* (dynamic) omniscient:

Sajoga-Kevalī - 21.
In general, there are dynamically omniscient living beings .

Q. What is the definition of *Sajoga-Kevalī* (dynamically omniscient) ?
A. The term *kevala* here indicates absolute knowledge or omniscience. The absolute knowledge is defined as the non-assisted knowledge which does not require the services of senses, mind and light. The omniscient is the individual who achieves this type of knowledge . The term *yoga* (activity) is the propensity of mind, speech and body. The individual having these activities is known as *sajogi.* The term activity is the terminal pointing here, and therefore, it indicates that all other precceding and lower stages including this stage also have activities. This spiritual stage has *kṣiṇa-bhāva* (destructional volition) due to the destruction of four destructive karma including *mohaniya* karma. It is also said in the *GJ,* verses 63 -63: "The eternal scriptures state that the individual is called [a] *kevalin,* because of his non-assisted infinite knowledge and

conation, [b] *sajogi* because of activities or dynamism and [c] Jina because he is devoid of destructive karma whose ignorantial darkness has been totally destroyed due to the rays of the sun of omniscience and who has been designated as supreme soul because of the manifestation of nine absolute prodigies of: destructional donation, gain, enjoyment of consumables and nonconsuimables, infinite potency, righteousness, omniscience, conation and conduct."

Now the venerable saint scholar Puṣpadanta states the next verse to describe the last and the fourteenth spiritual stage:+

Ajoga-Kevalī - 22.
In general, there are *ajoga* (static) *kevali jivās.*

Q. What is the definition of *ajoga kevali* ?
A. He is the kevalin with absolute knowledge and whose activity has become static.

Q. Which of the five *bhāvās,* the *ajoga kevali* possess ?
A. There is the *kṣayika bhāva* in this stage because of the total destruction of the four *ghātiya* (destructive) karmas* and because of their readiness to destroy the other four *aghātiya* (non-destructive) karmas.** It is said also in the *GJ,* verse 65: "The *ajoga kevali* are those [a] who have enriched themselves with 18, 000 types of good conduct; [b] have completely stopped the influx of new karma; [c] are devoid of new karmic bonds, and [d] are omniscience without activity."

**Āaāvaraṇīya, darśanāvarṇīya, mohanīya and antarāya. ** Nāma, āyuh, gotra and vedanīya.* + Thus there are fourteen spiritual stages leading from the volitioned state to the volitionless stage and from the lowest to the infinite internal energy. All these stages are associated with worldly living of the *jivās.*

Now Puṣpadanta describes *Siddhā* (spiritual stage) after describing the worldly fourteen stages:

Siddhā-cedi - 23.
In general, there are the *Siddhā jivās.*

Q. Who are the *Siddhā jivās* ?
A. They are those who have [a] destroyed all karmas [b] acquired the infinite bliss discarding worldly materials, [c] possessed all the best attributes, and [d] their soul is lesser than their terminal body, and [e] reside at the apex of the universe.

- to continue in forthcoming issues

BOOK REVIEWS

Absent Lord*: *Ascetics and Kings in Jain Ritual Culture. Lawrence A. Babb. New Delhi, Motilal Banarsidass. 1996. Pp. xi + 244, with glossary and index. ISBN 0-520-203240-0 (Softbound)

In his latest contribution Lawrence Babb creatively ruptures the bubble of *analytic structuralism* (Bell, *Ritual Theory,* 1992:219) and imaginatively illustrates how components of the *new ethnography* can assist his readers to grasp the subtle complexities contained within the "ritual theatre" of the *Mūrtipūjak* Jaina communities of Jaipur and Ahmedabad. Although this methodological approach to ritual might appear to be a stretch for some, he comes by it honestly and makes an excellent case -- by paraphrasing Milton Singer. According to Babb, there is, however, a notable difference between the "theatre of entertainment" and the "theatre of ritual." . That is, the players are largely their own audience, and their roles and performance are reliant on interactive discourse.

Babb raises two fundamential questions: Who are the performers? And what are their *roles* in the dramaturgy of their *ritual theatre*? Human performers are the obvious answer. Babb recognises in chapter 1, it is not just living beings who *perform* in ritual: objects of worship, but also play an important *role.* As he states, the dynamics operating within the *pūjās.* are not just a matter of interaction between the lay-community and the living *sādhus* and *sādhvis*.. There is also ritual interaction between the living *cast of characters* and non-living *other* such as the *Tīrthaṅkaras* (as both liberated soul -- the ascetic ideal -- and *mūrti* as *axis muni* of the community) and the *Dādāgurus* (the icons of the magical monks of antiquity).

According to Babb the worship of the *Tīrthaṅkaras* presents a significant existential problem for those *acting* within the ritual performance. Unlike the Hindu *pūjā* in which the worshiper anticipates the recovery of offerings that have been transformed by a divine receiver, the *Tīrthaṅkaras* are fully liberated beings who are no longer present cosmologically. He is quick to anticipate the obvious question: With the *mūrti* being both a non-living and non-present entity, what posible *meaning* and *expectation* lies behind such rituals as the *snātra pūjā* (bathing of the *mūrti*) or the *aṣṭprakāri pūjā* (eight-fold worship)? He

finds answers from his Jain informants: Jain layperson does not expect a response from the non-living *other*, but instead, sees ritual performance as both a self-reflexive act and a "substitute form of world renunciation . . . .the principal means of shedding the karmas that impede the soul's liberation." In this sense, he explains "the act of worshipping an ascetic becomes -- itself -- and ascetic act (92)."

Babb from his fieldwork in the temple complex at Mohan Bāṛī (Khartar *gaccha*) notes a ritual subculture that venerates the sacred *personae* of the *Dādāgurus*. Contrary to the ritual network involving the *Tīrthaṅkaras*, the *Dādāgurus* are deceased ascetic monks who have the ability -- much like the ritual complex involving a Hindu deity -- to respond to and within a ritual performance. In this context of the Jain veneration for asceticism, however, there is a crucial difference between the *ritual theatre* of the *Dādāgurus* and Hindu *pūjā*. As Babb states, "the *Dādāgurus* are beings who behave as deities, but because they are ascetics, belong to the category of beings who are (unlike deities) truly worthy of worship (134)." Or as we see in chapter 4, "these figures are Jain ascetics who can be worshipped as gods [and] who will advise their devotees in worldly matters. . . .they provide a method for lay Jains to pursue worldly values through ritual, but ritual of a sort that is nonetheless indisputably Jain (171)."

In conclusion then, what are Babb's readers to make of this Śvetāmbara ritual culture? On the surface, particularly in the case of the *Dādāgurus*, one is forced to ask: is this not a ritual culture that in many ways is similar to that of the Buddhist, Vaiṣṇavas and Śaivas? In a penetrating discussion that incorporates theory, tradition and conversations with his informants, Babb recognises that ritual reflexivity within the Jaina context may have "ritual crossovers" with the larger South Asian ritual culture. There is, however, an extremely important component that separates the Jaina tradition from that of "Puṣṭimārg's Kṛṣṇa and Raheja's godlings (192)." In the situation of any *ritual theatre* involving the *Tīrthaṅkaras*, there is both an ideal of asceticism and a non- *transactional* presence that is absent from the rituals of the Jaina's South Asian counterparts. As for the mixed case of the *Dādāgurus*, Babb concludes that despite the similarities to the worship of Hindu deities, the object of worship is still a Jain ascetic whose "historical context and supporting ideology are clearly Jain. . . .not a form of "Hinduism" grafted onto a Jain base (193)."

- Mikal A. Radford, a Doctoral student in the Department of Religious Studies and Social Sciences at McMaster University Canada.

Open Boundaries: Jain Communities and Cultures in Indian History. Editor, John E. Cort. Albany: State University of New York Press. 1998. Pp. vii + 264 with references and index. SUNY Series in Hindu Studies: Wendy Doniger, editor. ISBN 0-7914-3786-8 (Softbound).

In his critical analysis of the relationship between language and literary interpretation during the 1970s, Stanley Fish *comme* Foucault posits that the interpretive activities of a particular community should never be considered self-contained phenomena operating with a fixed language system. Instead, it should be thought of in terms of being an entity whose capacity to interpret meaning is interwoven within the fabric of an interactive social context (*Is There a Text in This Class*?).

A parochial community, therefore, derives meaning from the practices and assumptions of its own institutions, and to a varying degree, from those institutions that surround it. From this perspective, Fish continues, the meaning of either an utterance or text "is not a function of the values its words have in a linguistic system that is independent of context; rather, it is because the words are heard as already embedded in a context that they have a meaning (309)."

Pushing past the margins of literary criticism, the contemporary discourse over meaning, identity and context has solidly entered the parlance of ethno-historians, and is what lies at the heart of each of the ten essays that comprise *Open Boundaries: Jain Communities and Cultures in Indian History*. As John Cort states in his introduction:

> A sense of self-identity, whether in terms of the individual person or a social group, in never constructed in isolation, but rather is always a contextualized process, in which the sense of "self" is in dialogue, opposition, or dialectical relationship to a sense of what is "not-self" or "other.". . .Jains have always been active participants in larger contexts, and that therefore any adequate understanding of the Jains and Jainism must take into account both the larger contexts and the forms of Jain participation in those contexts (1-2).

Though admittedly not a new methodological approach, what makes this text superbly unique is its ground-breaking application to such a wide range of subject material (i.e., philosophy, the use of *mantra*, narratives, art, ritual, sex, politics, and issues of gender). Beginning with discussions held at the 1991 University of Wisconsin-Madison Conference on South Asia and a four-day workshop held in the summer of 1993 at Amherst College, it was agreed that for too long the Jaina tradition has been studied in both the West and Indian as either a

singular, isolated socio-religious entity that has been mistakenly separated from the context of a multiform South Asian landscape, an "essentially marginal, unimportant heterodox group (Buddhist-Jain)," or "a degenerationist model in which a supposedly pure, original ur-Jain doctrine is contrasted with the later impure, degenerated Jainism largely composed of half-understood and ill-digested Hindu influences and accretions (3)."

The intent of *Open Boundaries* is to correct this type of reductionist analysis. Though the granting of direction and degree of influence of one tradition on the other may vary from article to article, each of the contributors to this project counter the image of a single, marginalised Jaina identity. Instead, through a wealth of methodologies adopted from such sub-disciplines as textual analysis, history, art, economics, anthropology, sociology and ritual studies, this project has pieced together an extraordinary mosaic that portrays the Jainas as a varied, interactive and dynamic community involved in an energetic *discourse* not only with its own constituents, but with South Asian society as a whole.

In sum, the scholarship of John Cort, Christopher Key Chapple, Paul Dundas, Gary A. Tubb, James Ryan, Michael W. Meister, Lawrence A. Babb, Indira Viswanathan Peterson, Leslie Orr and Richard H. Davis has contributed to a project that is impressive, innovative, and more importantly, accessible to scholars, students of Jainism, and neophytes alike. Though at times the *boundaries* between what has been labeled *orthodox* and *heterodox tradition* can get disoncertingly blurred -- and this tends to be more a problem for those who have come to depend on static categories rather than a problem with the text itself -- this project is a marvelous foundation to what will hopefully become a new direction in both Jaina and South Asian studies. □

- Mikal A. Radford, a Doctoral student in the Department of Religious Studies and Social Sciences at McMaster University Canada.

Jaina Philosophy and Religion. Monk Nyayavijayaji. English Tr. by Nagin J. Shah. New Delhi, Motilal Banarsidass & B.L. Institute of Indology, 1998. pp.437. Price:Rs. 450. ISBN: 81-208-1490-8.

This is a voluminous work - a translation from Gujarati - encopasses the total philosophy of Jain religion in six chapters. The eastern philosophy always looked into the final end from life-cycle, and in this connection Jainism being one of the oldest religions dating back to the pre-advent of the Āryans has been said to be one of the foremost

structurally organized traditions that originated in the Indian subcontinent. The blissful happiness of mokṣa in Jainism is pivoted on the lives of all living beings and their individual path as envisaged by precepts of ahimsa, truth, karma, conduct and ethics. This theme was furthermore rejuvanated by Lord Pārṣva in ca.800 B.C.E. and by Lord Mahāvīra in ca. 600 B.C.E. In the later period Lord Buddha, a contemporary of Mahāvīra, set on the same goal sought the end to the misery of *samsāra*.

The first chapter deals with the to be liberated soul and its surrounding matter. It follows a chapter on the path and process for liberation. The third chapter deals with metaphysico-ethical and spiritual discussion, followed by a chapter which completely deals with the Jaina doctrine of karma and its mechanism. Chapter five: Jaina Logic presents *syādvāda* doctrine supported by *saptabhṅgi* (sevenfold judgement) and *naya* (standpoint). The final chapter underlines the very nature of Jaina secular philosophical approach quoting and enumerating ancient texts and saint scholars - Haribhadra, Hemacandra, Yasovijaya and Jayasekharasuri.

For students of religion or for ordinary folks, this book is very helpful for it presents a comprehensive Jaina philosophy very authoritatively. This could have its title as Basics of Jaina Philosophy.

- Ashij J. Kumar, a Graduate student in the Department of Environmental Sciences at University of Toronto, Canada.

Ācārānga: Restoration of Śrutaskandha Chapter One. Editor. Dr. K.R. Chandra. Ahmedabad, Prākrit Jain Vidhya Vikas Fund, 1998.

The *Ācāranga*, first canonical text of the Jainas, has been examined and discussed by Dr. K.R. Chandra, Professor Emeritus of Prākrit, as to its language and character. Conceptualizing and applying the principles of restoration of the original language of ancient Ardhamāgadhi, Chandra has produced this work on the basis of available archaic readings from the manuscripts. Substantiated by evidence, he has produced a glimpse of some phonological and morphological features of the text.

The work is indeed an important contribution to the study of phonology, morphology and orthography of Prākrit language.

- Ashij J. Kumar, a Graduate student in the Department of Environmental Sciences at University of Toronto, Canada.

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