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Historical Notes

The Jaina School of Indian Mathematics*

Dipak Jadhav**

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Abstract

The Jaina school of Indian mathematics had a considerable standing. The school, on the basis of theorization, could be divided into the canonical class and the exclusive class. The treatises of the former contain mathematics along with discussion on Jaina canons while those of the latter are composed exclusively on mathematics. The object of the former was to demonstrate canonical thoughts including on *karma* and cosmos using mathematics while that of the exclusive class was to provide mathematics education to the contemporary civil life. Besides *laukika gaṇita* (worldly mathematics) and *lokottara gaṇita* (post-worldly mathematics) the paper also addresses some related issues.

Key words: Canonical class, Exclusive class, The Jaina school of Indian mathematics.

1. Introduction

The part played by the Jainas in the development of mathematics in ancient and medieval India is very significant. In accordance with their requirements they followed their own line of adoption and development of mathematics, which is generally called the Jaina school of mathematics. It, being developed within the intelligentsia of Indian mathematics, is, hereupon and in his previous papers[1], termed "the Jaina school of Indian mathematics" by the present author.

The school is suggested to have been divided into the canonical class and the exclusive class. A large number of papers[2] emphasizing the canonical class and the exclusive class have already been reported by the present author before. He first elaborated them in his paper appeared in 2004 (Jadhav 2004, p. 37), and thereafter in his doctoral thesis (Jadhav 2013, pp. 34-48). In this paper we are going to deal with them in detail.

However, there is a general impression, especially outside India, that there was not any organized school of mathematics in ancient and medieval India except the Kerala school of astronomy and mathematics. On the Jain school of Indian mathematics there is a general concern, again outside India, whether Jain mathematics should be treated as a school rather than a specific tradition, closely related to mainstream of classical Sanskrit mathematical writings, and why the mathematicians like Śrīdhara and Mahāvīra be

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**Lecturer in Mathematics and Principal-in-charge, Govt. School of Excellence, Barwani (M. P.) India. E-mail: dipak_jadhav17@yahoo.com

[1] Jadhav 2003, p. 53; 2004, p. 37; 2006, p. 75; 2008, pp. 139 and 146-147; 2009, p. 52; 2014, pp. 260-261 and 263-265; Jadhav and Jain 2003, p. 91; Jadhav and Padmavathamma 2002, p. 31.

[2] Jadhav 2001, p. 94; Jadhav and Padmavathamma 2002, pp. 51-53; Jadhav 2002b, pp. 251-252; Jadhav and Jain 2003, pp. 108, 113-117; Jadhav 2004, p. 37; 2006, p. 75; 2008, p. 139; 2013, pp. 140 and 146-147; 2014, pp. 260-261; Jadhav and Jain 2016, pp.190-204

included into exclusive class rather than as members of the mainstream classical Sanskrit mathematical tradition, who simply happen to be Jainas.

The present paper is mainly aimed at justifying and discussing the division of the Jaina school of Indian mathematics into the canonical class and the exclusive class. This will be done by means of theorization. In this regard, a set of factual ideas will be developed about the school in order to find some basis. By studying the way in which its treatises contain mathematics along with canonical discussion or exclusively and its mathematicians treat, we will explain how it fits into the above classes. In order to avoid any misunderstanding regarding the exclusive class against *laukika gaṇita* (worldly mathematics) and the canonical class against *lokottara gaṇita* (post-worldly mathematics) an appropriate discussion containing a comparative analysis of these two different divisions has been accommodated.

2. Approval of the Jaina school of Indian mathematics

Every academic discipline, from old theology to modern science and technology, has competing theories and perspectives with which it grows. Mathematics has been no exception. For example, John Napier (1550-1617 CE) and Jobst Bürgi (1552-1632 CE) discovered logarithms, but through an entirely different line of approach. The former's approach was geometric while the latter's was algebraic. Long before them the Jaina school of Indian mathematics approached logarithms on the basis of the number of possible divisions of a quantity by two (Jadhav 2002a; Jadhav 2003; Jadhav 2014).

The studies made on mathematical thoughts developed in ancient and medieval India and about their followers make us to appreciate that the schools of some sort did exist. David Eugene Smith, in the introduction written by him to the *Gaṇita-sāra-saṅgraha* ('Compendium of the essence of mathematics') of Mahāvīra (c. 850) published in 1912, opines that:

> "The answer to the questions as to the relation between the schools of India cannot yet be easily given. At first it would seem a simple matter to compare the treatises of the three or four great algebraists and to note the similarities and differences. When this is done, however, the result seems to be that the works of Brahmagupta, Mahāvīrācārya and Bhāskara may be described as similar in spirit but entirely different in detail. For example, all of these writers treat of the areas of polygons, but Mahāvīrācārya is the only one to make any point of those that are re-entrant. All of them touch upon the area of a segment of a circle, but all give different rules. The so called *janya* operation is akin to work found in Brahmagupta and yet none of the problems is the same. The shadow problems, primitive case of trigonometry and gnomonics, suggest a similarity among these three great writers and yet those of Mahāvīrācārya are much distinct than the one to be found in either Brahmagupta or Bhāskara and no questions are duplicated (Padmavathamma, 2000, p. 762)."

Smith accepts as early as in 1912 in more or less clear terms that there were the schools of mathematics in ancient and medieval India.

In ancient India, mathematics was not separated from astronomy. In fact, the former was developed for the service of the latter. It is now recognized that there was Brāmapakṣa in Indian classical mathematical astronomy or Bramagupta school of Indian astronomy after the name of Indian mathematician and astronomer Brahmagupta (628 CE) (Plofker, 2014).

As far as the Jaina school of Indian mathematics is concerned, we shall see that it sustained for more than two thousand years adopting, developing, following and practicing certain kinds of mathematical thoughts in ancient and medieval India.

Bibhutibhusan Datta appears to be the first historian of mathematics who wrote a paper of which title contains the name of the school. The paper, which he wrote in 1929, is "The Jaina school of mathematics" (Datta 1929). It was aimed at professing, although he did not claim so, an account of the mathematical achievements of the Jainas. It was based on those sources that he collected up to that time. Most of them were of the *Śvetāmbara* tradition. In 1934, S K Das wrote "The Jaina school of astronomy" (Das 1934). It gave the details of the astronomical and cosmographical speculations of the Jainas. L C Jain produced four papers. The first was "On the Jaina school of mathematics" (Jain, LC, 1967). It was also aimed at professing a brief account of the mathematical achievements of the Jainas but the sources that he used were different from those used by Datta. Those sources were of the *Digambara* tradition. "Jaina school of mathematics" was the second one which he produced (Jain, LC, 1975). It was a study in Chinese influences and transmissions. The third one was "The Jaina schools of mathematical sciences" (Jain, LC, 1992). It describes that there have been two Jaina schools of mathematics in India, the *Digambara* and the *Śvetāmbara*. The former held proficiency in the symbolico-mathematical theory of *karma*, whereas the latter seems to be more interested in astronomy and astrology. "The Jaina school of mathematical philosophy" was the fourth paper (Jain, LC, 2000) which discussed how the Jainas approached their philosophy all the way through mathematics.

Apart from the above four papers, L C Jain wrote five more papers that contain the expression "Jaina school of mathematics" in their respective titles, namely (1) "Set theory in Jaina school of mathematics" (Jain, LC, 1973), (2) "On certain mathematical topics of the *Dhavalā* texts (the Jaina school of mathematics)" (Jain, LC, 1976), (3) "Perspective of system-theoretic technique in Jaina school of mathematics between 1400-1800 CE" (Jain, LC 1978), (4) "System theory in Jaina school of mathematics" (Jain, LC, 1979), and (5) "System theory in Jaina school of mathematics II" (Jain, LC and Jain, Meena 1989).

Despite all these publications, the expression "the Jaina school of mathematics" did not get an essential amount of exposure at international level. One of the reasons behind this situation may have been that the above papers were published in less known journals except in *Indian Journal of History of Science*. Another may be that it was not recognized that the mathematicians of Jaina faith shared common mathematical thoughts to a great extent irrespective of the languages they used to compose their treatises. For example, (1) the Jainas shared $\sqrt{10}$ as the value for π for the long period commencing from 500 BCE at least to the time of Toḍaramala (1720 CE–1767 CE) (Jadhav, 2013, pp. 502-517 and 528-538), and (2) Ṭhakkura Pherū borrowed most of the rules into the *Gaṇita-sāra-kaumudī* ("Moonlight of the essence of mathematics") composed by him in Apabhraṃśa from the *Triśatikā* composed by Śrīdhara in Sanskrit (see SaKHYa 2009).

With the assessment of the situation R C Gupta approves, while writing a note on the research work done by L C Jain, the expression "the Jaina school of mathematics" in the following words.

> "The Jaina school of mathematics was one of the most remarkable institutions of ancient India. Its contribution in the development of scientific thought especially as part of philosophic-mathematical thinking may be regarded as quite significant and is a known fact to some extent. But the paradoxical situation is that it is yet to find due place in the historical expositions of the development of mathematics in India, what to say of that in the world. Nevertheless, in the pursuit of scientific thinking the depth of Jain philosopher-mathematicians is comparable to that of Greece. For

> example, they were the earliest to transcend the simplistic thesis that all infinities are equal (Gupta 1991, p. 88)."[3]

In fact, the number of truly devoted research scholars in the field of the study of the Jaina school of Indian mathematics is relatively very small. There are certain difficulties which are responsible for this situation. According to R C Gupta, some of them are difficulties of ancient languages and peculiar terminology, of historical as well as scientific methodology, and of other technicalities involved in the Jaina texts (Gupta 1992, p. xi). One more difficulty Kim Plofker, while putting a remark of appreciation on the research work done by R C Gupta[4], maps out in the following words.

> "Gupta has published several key papers on the remarkable mathematical discoveries of the Jaina tradition; this has been a yeoman service especially in the case of the many works that have been almost inaccessible to anyone not closely linked with the Jaina canon (Plofker 2009b, p. 116)."

Despite the aforesaid difficulties the Jaina school of Indian mathematics is a fascinating field of ancient and medieval Indian mathematics to be explored.

3. The Division of the School into the Canonical class and the Exclusive class

The Jaina school of Indian mathematics is said to have initiated its activity in the time of Lord Ṛṣabha, the first and foremost *Tīrthaṅkara* in the history of Jainism. He is said to have taught numbers to his daughter Sundari with his left hand from right to left. This is communicated in the *Ādīpurāṇa* of Jinasena as late as in 9th century (Jain, Pannalal 1993, v. 108, p. 356).[5] It is, on the basis of this communication, not easy to corroborate that the school took, as Lord Ṛṣabha belongs to the prehistoric period[6], its initiation in his time. However, this is only meant to suggest the antiquity of the school. However, he is well known to the Vedic literature.[7] Hence, according to T A Sarasvati,

> "it is more likely that this dissident faith ‹i. e., Jainism›, revolting against sacrificial killing, was quite an old rival to the Vedic faith or that it had taken root in India even before the Vedic faith. The mathematical knowledge contained in the Jaina writings should therefore have been more or less parallel to that in the Vedic literature (Sarasvati 1979, p. 61)."

On the other hand, according to L C Jain, the school appears to have originated soon after Lord Mahāvīra (599 BCE-527 BCE), the twenty fourth and last *Tīrthaṅkara* in the history of Jainism. It means that the school flourished in India around the same period in which the school of Pythagoras (572 BCE-510 BCE) flourished in Greece. It was formed mainly of some *niggaṃṭha* (Skt. *nirgrantha*, outwardly and inwardly free from worldly ties) ascetics who left a few record of their knowledge (Jain, L C 1967, p. 265).

The present author, however, believes that the Jaina school of Indian mathematics was in cradle prior to Lord Mahāvīra, if existed. It started to flourish from Lord Mahāvīra's own time and came into black and white a little long after him. And it developed along with the progress and expansion of Jainism in India.

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3 For the details regarding the example cited in this passage, see Singh, Navjyoti 1991, p. 229.

4 For the research work done by R C Gupta, see Hayashi 2011.

5 Also see Jain, Anupam 1994, p. 127 where it is also stated that there are the other texts such as the *Bhagavatī Sūtra*, the *Purāṇasāra Saṃgraha* of Dāmanandi*, the Śatruñjaya Kāvya* etc that document that Sundari learnt mathematics from her father.

6 Johar 2000, pp. 46-51. Also see Jain, H. L. 2000, pp. 3-28; Kumar 1997, pp.44-45; McEvilley 1996, pp. 6-20; Ranga 2000, pp. 73-75.

7 Johar 2000, pp. 46-51. Also see Jain, H. L. 2000, pp. 3-28; Ranga 2000, pp.73-75.

The teachings of the last *Tīrthaṅkara* were systematized by their disciples into doctrinal theories called canons (*āgamas* or *siddhāntas*) or sacred scriptures (*śrutis*). *Āgamas* refer to 'that which have come down' to us from Lord Mahāvīra. They are called *siddhāntas* because they are the fundamentals of his speeches. According to the unanimous tradition of the *Digambara* and *Śvetāmbara* Jainas, the teachings of Lord Mahāvīra were arranged in twelve Books called *aṅgas* by his disciples and successors; each *aṅga* has been called a *sutta* which is sanskritized as *sūtra*. Those twelve *aṅgas* including the *Thāṇa* (Skt. *Sthānāṅga*) and the *Vikkhāpaṇṇatti* or *Viyāhapannatti* (Skt. *Vyākhyāprajñapti* or better known as the *Bhagavatī ‹Sūtra›*) together formed the earliest literature on Jainism (Jaini 1927 vv. 356-359, pp. 202-203 and Schubring 2000, p. 80; also see Jain, J. P. 1979, p. 8). The *Śvetāmbaras* hold that the first eleven *aṅgas* have come down to us as they were thought in a much curtailed and revised form. Only the twelfth *aṅga* has been lost. On the other hand, the *Digambaras* do not accept this tradition. According to them, the whole of the original canon was preserved only for 165 years after Lord Mahāvīra up to his eighth successor. Later its portions began to be gradually lost. What had remained of it for 683 years after him was fragmentary. It has been reproduced by subsequent writers in their own language (Jain, H L 2000, pp. 34-35 and 41). The *Samayasāra*, the *Pañcāṣṭikāyasāra* etc composed by Kundakunda during some period between 100 BCE and 100 CE, the *Kaṣāya Pāhuḍa* written by Guṇadhara during some period between 10 BCE and 25 CE, and the *Ṣaṭkhaṇḍāgama* written by Puṣpadanta and Bhūtabalī during some period between 87 CE and 156 CE are the earliest available canonical literature amongst them. The post-canonical works were also written by the Jainas, especially by the *Digambaras*. They were composed from the fifth century CE to the eleventh century CE. They mainly deal with *karma* theory; something with cosmology and cosmography. The *Dhavalā* of Vīrasena (816 CE), the *Jaya Dhavalā* of Jinasena (9th century CE), the *Gommaṭasāra* (*Jīvakāṇḍa* and *Karmakāṇḍa*), *Labdhisāra* (inclusive of *Kṣapaṇāsāra*), and *Trilokasāra* of Nemicandra (981 CE), *Pañcasaṅgraha* of Amitagati (11th century CE) are amongst their post-canonical literature. For over 2000 years, different truth-seekers, especially the Jaina ascetics, have propagated Jainism in India in different ways. They codified their canonical literature using Prakrit. The forms of Prakrit that were used by them were *Ardhamāgadhī*, *Apabhraṃśa*, *Śauraseni* and *Jaina Mahārāṣṭri*. Meanwhile, they also compiled their literature in Sanskrit, Kannaḍa, etc. The *Śauraseni* Prakrit is considered to be the representative language of the *Digambara* Jaina literature whereas *Ardhamāgadhī* to be that of the *Śvetāmbara* Jaina literature. The Jaina ascetics from both the sects had been writing suggestions (*prajñaptis*) on, compendiums (*saṅgrahas*) of, and essences (*sāras*) of their canons, and commentaries thereon until 1800 CE. Even they have written in later than 1800 CE. The literature of the Jainas is thus very vast and varied. The discussions on cosmology and *karma* theory form the most important part of their literature.

According to an ancient fourfold classification of the literature on the Jaina canons, one is the *Karaṇānuyoga* ("discipline of manuals") attributed to the *Digambaras* or the *Gaṇitānuyoga* ("discipline of mathematics") attributed to *Śvetāmbaras*.[8] The classification shows that the Jainas took keen interest in the study of mathematics and attached great importance to the culture of mathematics. And this discipline of science was regarded as an integral part of their religion. The knowledge of it was considered to be one of their principal accomplishments. For

[8] Jain, Anupam 2008, pp. 6-7. He has given the reference of '*Ratnakaraṇḍa Śrāvakacāra* of Ācārya Samantabhadra, vv. 2.43-2.46' for the Digambaras' classification and the reference of '*Āvaśyaka Kathā*, śloka 174' for the Śvetāmbaras' classification.

over 2000 years the Jainas adopted, developed, followed and practiced certain kinds of mathematical thoughts as a school in ancient and medieval India. It was very vast and wide.

Those treatises that are on the Jaina canons and contain mathematics for their discourse are placed in the canonical class. For example, the *Bhagavatī Sūtra* refers to minimum number of points (*paesas*, Skt. *pradeśas*) required to construct each of the eleven formations as it was essential for the discussion therein. Distinction has been drawn between odd and even number of points. Those numbers have been recognized as figurate numbers (Jadhav 2009, pp. 35-55). The treatises that have been composed by the authors of Jaina faith exclusively on mathematics are placed in the exclusive class. The contents in the treatises of this class are cent per cent mathematical ones. For example, the well known treatise of this class is the *Gaṇita-sāra-saṅgraha* of Mahāvīra (c. 850 CE).

The division of the Jain school into these two classes is based on factual ideas and may be appreciated as explianed below separately:

3.1. Canonical class

The authors of this class were not mathematicians alone. They were the authors of the canonical or the post-canonical works. In other words, they were the authors of philosophy or ontology or metaphysics or cosmology or *Karma* theory or any combination of them oriented treatises. Historians of mathematics put them into the rank of mathematicians as there is found a good deal of knowledge of mathematics in their treatises. Mathematical material found embedded in their treatises seem to have been developed or dealt by them in accordance with their need and was accurately applied on cosmological system or *Karma* system or used for some philosophical discussion. It is available in abundance. It sometimes occurs in the form of rules and results in their treatises and every so often occurs in the functional form. For example, the laws of logarithms to the base two (Jadhav, 2002a) and combinatorics of tuples (Jadhav and Jain, 2016) are available in the form of rules in the *Trilokasāra* and the *Gommaṭasāra* (*Jīvakāṇḍa*) respectively whereas system theory of its own kind is available in the functional form (Jain, L C 1979; 1989; Gupta, 1993, p. 24).

Some of the mathematicians of this class are listed in Table 1. This does not however profess to be a complete list of the mathematicians of this class. In fact, it is a small list that covers only the prominent mathematicians.

3.2. Exclusive class

The treatises of this class are exclusively written on mathematics. The subject matter of their treatises is mathematics and only mathematics. It usually, but not at all times, happens to be a complete course on arithmetic and mensuration to cater the needs of both students and civil activities. The authors of this class were originally mathematicians except for few ones. Some of the mathematicians of this class are listed in Table 2, however it does not profess to be a complete list of the mathematicians of this class. In fact, it covers only the prominent mathematicians.

Unlike in the treatises of the canonical class except the *Sthānāṅga sūtra* (Madhukara 1992, Chapter X, *sūtra* 100, p. 720) that suggests the ten topics for discussion in *saṃkhyāna* (computation) and the *Trilokasāra-Bhāṣāṭīkā*[9] of Ṭoḍaramala (1720–1767 CE) (Sastri, Manoharalal 1918, pp. 1-22, and Bharilla 1999, p. 104) and like in those of the non-Jaina mathematicians such as in the *Brāhma-sphuṭa-siddhānta* of Brahmagupta (628 CE), the subject matter in the treatises of the exclusive class is broadly divided into two sections. One is *parikarma* (logistics) and

[9] It is inexactly written the *Bhāṣā Vācanikā*. See Bharilla 1999, pp. 101-102.

Table 1: Canonical class

S. No.	Mathematician	Sect	Major Works	Written in
1	Anonymous (c. 500 BCE)	*Śvetāmbara*	*Sūrya Prajñapti* (Madhukara 1995)	Prakrit
2	Anonymous (c. 500 BCE)	*Śvetāmbara*	*Jambūdvīpaprajñapti* (Sastri, Chhaganlal 1994)	Prakrit
3	Anonymous (c. 500 BCE)	*Śvetāmbara*	*Jīvājīvābhigama Sūtra* (Madhukara 1989, Part I and 1991, Part II)	Prakrit
4	Anonymous (c. 300 BCE or earlier)	*Śvetāmbara*	*Uttarādhyayana Sūtra* (Madhukara 1991, Publication No. 19)	Prakrit
5	Anonymous(c. 300 BCE)	*Śvetāmbara*	*Sthānāṅga Sūtra* (Madhukara 1992)	Prakrit
6	Sudharma Svāmī (c. 300 BCE or earlier)	*Śvetāmbara*	*Bhagavatī Sūtra* (Deleu 1970)	Prakrit
7	Puṣpadanta and Bhūtabalī (between 87 CE and 156 CE)	*Digambara*	*Ṣaṭkhaṇḍāgama* (Jain, H. L. et al.1996)	Prakrit
8	Umāsvāti /Umāsvāmī (between 150 BCE and 219 CE)	*Śvetāmbara/ Digambara*	*Tattvārthādhigama Sūtra Bhāṣya* (Sūrīśvara 1994)	Sanskrit
			Jambūdvīpa Samāsa (Śrīsatyavijaya 1923)	Sanskrit
9	Āryarakṣita(3rd century CE)	*Śvetāmbara*	*Anuyogadvāra Sūtra* (Madhukara 1987)	Prakrit
10	Anonymous Vallabhīcārya (c. 300 CE)	*Śvetāmbara*	*Jyotiṣkaraṇḍaka* (Anonymous 1928)	Prakrit
11	Yativṛṣabha (between 176 and 609 CE)	*Digambara*	*Tiloyapaṇṇatti* (Patni 1997)	Prakrit
12	Jinabhadra Gaṇī (609 CE)	*Śvetāmbara*	*Bṛhatkṣetrasamāsa* (Vijayaji 1988)	Prakrit
13	Vīrasena (816 CE)	*Digambara*	*Dhavalā* (Jain, H. L. et al. 1996)	Prakrit
14	Nemicandra (981 CE)	*Digambara*	*Trilokasāra* (Mukhtara and Patni 1975)	Prakrit
			Gommaṭasāra (Jain, G. L. and Jain, S. L. 1919)	Prakrit
			Labdhisāra (Jain, G. L. and Jain, S. L. 1919)	Prakrit
15	Mādhavacandra Traividya (c. 982 CE)	*Digambara*	Commentary on the *Trilokasāra* (Mukhtara and Patni 1975)	Sanskrit
16	Padmanandi (1000 CE)	*Digambara*	*Jambūdvīpapaṇṇattisaṃgaho* (Upadhye and Jain 1958)	Prakrit
17	Abhayadeva Sūri (1015-1078 CE)	*Śvetāmbara*	*Vṛttis* (commentaries) on the the *Bhagavatī Sūtra*, *Sthānāṅga Sūtra* etc (Jain, Anupam 2008, pp. 45-47)	Sanskrit
18	Malayagiri (1080–1172 CE)	*Śvetāmbara*	Commentaries on the *Jyotiṣkaraṇḍaka* etc (Anonymous 1928)	Prakrit
19	Vinayavijaya Gaṇī (1639 CE)	*Śvetāmbara*	*Lokaprakāśa* (Vijayaji 1932)	Sanskrit
20	Toḍaramala (1720–1767 CE)	*Digambara*	*Samyakjñānacandrikā* (Jain, G. L. and Jain, S. L. 1919)	*Ḍhūṇḍhārī*
			Trilokasāra-Bhāṣāṭīkā (Sastri, Manoharalal 1918)	*Ḍhūṇḍhārī*

the other is *vyavahāra* (determination). The *parikarma* (logistics or operation) consists of fundamental arithmetic operations and other important ways of computation. The *vyavahāra* (determination or practice or procedure) shares the topics of common concern.

The topics suggested in the *Sthānāṅga Sūtra* are *parikarma*, *vyavahāra*, *rajju*, *rāśi*, *kalāsavarṇa*, *yavat tavat*, *varga*, *ghana*, *vargavarga*, and *kalpa*.[10] Brahmagupta offers twenty *parikarmas* and eight *vyavahāras*. According to Pṛthudakasvāmī (c. 850 CE), a commentator of the *Brāhma-sphuṭa-siddhānta*, *saṅkalita* (addition), *vyavakalita* (subtraction), *guṇana* (multiplication), *bhāgahāra* (division), *varga* (square), *vargamūla* (square root), *ghana* (cube), *ghanamūla* (cube root), *pañca jāti* (five rules of reduction relating to the five standard forms of fractions), *trairāśika* (the rule of three), *vyasta-trairāśika* (the inverse rule of three), *pañcarāśika* (the rule of five), *saptarāśika* (the rule of seven), *navarāśika* (the rule of nine), *ekādaśarāśika* (the rule of eleven), and *bhāṇḍapratibhāṇḍa* (barter and exchange) are those twenty *parikarmas* and *miśraka* (mixture), *śreḍhī* (progression or series), *kṣetra* (plane figures), *khāta* (excavation), *citi* (stack), *krākacika* (saw), *raśi* (mound), and *chāyā* (shadow) are those eight *vyavahāras* (Datta and Singh 1935, p. 124).

Śrīdhara (c. 799 CE) tenders twenty nine *parikarmas* and nine *vyavahāras*. He excluded *ekādaśarāśika* (the rule of eleven) from the twenty *parikarmas* offered by Brahmagupta and offers the other expression *pratyutpanna* for *guṇana* (multiplication). The ten *parikarmas* added by him are the first eight *parikarmas* for *bhinnas* (fractions), one more rule of reduction relating to one more standard form of fraction, and *jīva-vikraya* (sale of living beings). One *vyavahāra* added by him to the list of eight *vyavahāras* offered by Brahmagupta is *śūnya-tatva* (principle of zero) (Shukla 1959, vv. 2-6, p. 2).

The exclusive class was aimed at providing mathematics for the sake of worldly business. In other words, the object of this class was to provide mathematics education to common people according to the necessity of the contemporary civil life. This can be corroborated from the statements of the mathematicians of this class. In the beginning of the *Triśatikā* Śrīdhara states that:

नत्वा जिनं स्वविरचितपाट्या गणितस्य सारमुद्धृत्य।
लोकव्यवहाराय प्रवक्ष्यति श्रीधराचार्यः।।

natvā jinaṃ svaviracitapātyā gaṇitasya sāramuddhṛtya|
lokavyavahārāya pravakṣyati Śrīdharācāryaḥ||[11]

> "Paying homage to *Jina*, having excerpted the essence (*sāra*) of mathematics (*gaṇita*) from the *Pāṭī*‹*gaṇita*› (algorithms) composed by himself, the teacher (*ācārya*) Śrīdhara will state ‹it› for the sake of worldly business."

And the *Pāṭīgaṇita* of Śrīdhara as well is aimed at providing mathematics *lokavyavahārārtha* ('for the sake of worldly business') (Shukla 1959, v.1, p.1). Rājāditya (12th century CE) claims that with a view to support all the scholars in the field of mathematics and help businessmen and common men better deal with their day to day transactions he wrote the *Vyavahāra-gaṇita* (Padmavathamma et al 2013, vv. 1.12-1.14, pp. 5-6). For the sake of all people Ṭhakkara Pherū expounded the *Gaṇita-sāra-kaumudī* after he had taken some material from the writings of the ancient teachers, especially from Mahāvīra (SaKHYa 2009, pp. xx-xxi) and Śrīdhara (SaKHYa 2009, pp. xxi-xxii), gained some from direct experience, and heard some from others (SaKHYa 2009, v. 1.2, pp. 9 and 45).

Śrīdhara's faith whether he was a Śaiva follower or a Jaina has been a great deal of controversy among scholars (Shukla 1959, Introduction, pp. xv and xxxv-xxxvi). The benediction referred to above is from an old palm-

[10] For their interpretations see Rajgopal 1991, pp. 1-8.
[11] See Sastri, N. C. Jain 1947, p. 31.

Table 2: Exclusive class

S. No.	Mathematician	Sect	Major Works	Written in
1	Śrīdhara (c. 799 CE)	*Digambara*	*Pāṭīgaṇita* (Shukla 1959)	Sanskrit
			Triśatikā (Dvivedi 1899)	Sanskrit
2	Mahāvīra (c. 850 CE)	*Digambara*	*Gaṇita-sāra-saṅgraha* (Padmavathamma 2000)	Sanskrit
3	Mādhavacandra Traividya (c. 982 CE)	*Digambara*	*Ṣaṭtriṃśikā* (Jain, Anupam 1982; 1988c)	Sanskrit
4	Rājāditya (12th century CE)	*Digambara*	*Vyavahāra-gaṇita* (Padmavathamma et al 2013)	Kannaḍa
5	Siṃhatilaka Sūri (13th Century CE)	*Śvetāmbara*	*Gaṇita-tilaka-vṛtti* (Kapadia 1937)	Sanskrit
6	Ṭhakkara Pherū (c. 1265-c.1330 CE)	*Śvetāmbara*	*Gaṇita-sāra-kaumudī* (SaKHYa 2009)	Apabhraṃśa Prakrit
7	Anonymous	-	Pāṭaṇa Mathematical Anthology (Hayashi 2006a)	Sanskrit
8	Hemarāja (c. 1673)	*Digambara*	*Gaṇitasāra* (Jain, Anupam 1988b)	Hindi
9	Tejasiṁha Sūri (died in 1686)	*Śvetāmbara*	*Iṣṭāṅka Pañcaviṁśatikā* (Hayashi 2006b)	Sanskrit

leaf manuscript of the *Triśatikā*, written in Kanarese script, discovered in the Jaina Library at Mūḍabidrī in south Karnataka. It contains '*Jinaṃ*' whereas the other manuscript contains '*Śivaṃ*' (Dvivedi 1899, p. 1). N C Jain Sastri is of the opinion that the occurrence of the reading '*Śivaṃ*' is a deliberate change as such a custom of changing the benediction of a text is found in other texts too. He regards the reading '*Jinaṃ*' to be authentic. So he suggests that Śrīdhara was a Jaina (Sastri, N C Jain 1947, pp. 31-32). Anupam Jain and Jaychand Jain support N C Jain with scores of arguments (Jain, Anupam and Jain, Jaychand 1988, pp. 49-53). Mamata Agrawal has followed them (Agrawal 2001, pp. 41-43).

4. *Laukika* and *lokottara gaṇita*

The mathematics found in the treatises of the Jaina school of Indian mathematics is viewed into two categories. One is *laukika gaṇita* (worldly mathematics) and the other is *alaukika gaṇita* (non-worldly mathematics) or *lokottara gaṇita* (post-worldly mathematics). These expressions were frequently used by L C Jain (Jain, L C 1961, pp. 222-231; 1973, p. 3; 1980, p. 43; 2007, p. 9). R C Gupta understands them in the way as follows.

> "The *lokottara* type of Jaina mathematics is somewhat of abstract and its higher level surpasses that of *laukika* mathematics. The *laukika* Jaina mathematics is mostly mensurational and is related to simpler problems of the type which we come across in ordinary life. It is covered by what we call elementary arithmetic, algebra and geometry. … It is in the category of *alaukika* mathematics that the work of the Jaina School is unique. In fact, the remarkable achievement in this area clearly distinguishes the Jaina school of Mathematics from other ancient schools whether it is in India, or outside India. One is often surprised to find parallels of several modern mathematical concepts and notions in ancient Jaina texts (Gupta 1993, pp. 22-23)."

The two expressions, *laukika gaṇita* and *lokottara gaṇita*, were rarely employed in the treatises of the Jaina school of Indian mathematics.

It is Ṭoḍaramala who made use of the expression *alaukika gaṇita* in the *Bhāṣāṭīkā*, an introduction to the mathematics appeared in the commentary of Mādhavacandra Traividya on the *Trilokasāra* of Nemicandra (Sastri, Manoharalal 1918, pp. 1-22; also see Jain, L C and Trivedi, R K 1987, p. 365 and Bharilla 1999, p. 102). Long before him the expression *laukikagaṇita* was employed, in order to show how the rule for finding the meeting time of the sun and the moon can be applied on a worldly problem, by Bhāskara I (629 CE) outside the Jaina school of Indian mathematics (Shukla 1976, p. 131; also see Keller 2006, p. 127). Here it may be clearly concluded that mathematics on its own was not *laukika* (worldly) or *lokottara* (post-worldly) but because of its application it was considered to be *laukika* or *lokottara*.

Bhāskara I (629 CE) seems to have been in contact with the treatises of the Jainas as he is found to have quoted five passages in Prakrit *gāthās* (verses) in the *Āryabhaṭīya-bhāṣya*. Two of them state the following two formulae. (1) $C \approx \sqrt{10d^2}$ where C is the circumference of a circle of diameter d, and (2) $A \approx \sqrt{10}\ c(h/4)$ where A, c and h are the area, chord and height of a segment of a circle of diameter d. The value for π implied in these formulae is $\sqrt{10}$, which was then regarded as the subtle one. His purpose of touching upon the passages was to criticizing $\sqrt{10}$ and emphasizing on Āryabhaṭa's new value $\frac{62832}{20000}$ (Shukla 1976, p. lvi; also see below v. 2.10, pp. 72-73). He seems to have cited those five passages from some contemporary or earlier work. That work, according to B B Datta, must have been of Jaina authorship (Datta 1936, p. 41). It may be noted that $\sqrt{10}$ continued to be used by one and all in the school as the value for π for the long period of more than two thousand years commencing from 500 BCE. The mathematicians of the canonical class except Vīrasena (816 CE) and Nemicandra (981 CE) never used any other value for π (Jadhav 2013, pp. 502-517 and 528-538).

The two adjectives, *laukika* and *lokottara*, do occur in the treatises of the school but with the term *māna* or *pramāna* (measure). The *Trilokasāra* refers to two kinds of *māṇa* (Skt. *māna*, measure), *logiga* (Skt. *laukika*, worldly) and *loguttara* (Skt. *lokottara*, post-worldly). The *laukika māna* is of six types while *dravyamāna* (fluent-measure), *kṣetramāna* (space-measure), *kālamāna* (time-measure) and *bhāvamāna* (thought-measure) are the four *lokottara mānas* (Mukhtara and Patni 1975, vv. 9-10, pp. 12-13). The *Tatvārthavārtika* ("Explanatory of the meaning of the fundamental principles") of Akalaṅka (7th century) appears to be the first treatise that contains the classification of measure in this manner (Jain, Mahendra 1999, *sūtra* 3.38, pp. 205-209).

One of the six *laukika mānas* is *gaṇi māna* (or *gaṇanā māna* or *gaṇima māna*, counting-measure). One, two, three and so on are counting-measures (Jain, Mahendra 1999, sūtra 3.38, p. 205). Salaries, wages, provisions, income, expenditure, cost etc are, according to the *Anuyogadvāra Sūtra*, determined using it (Madhukara 1987, *sūtra* 327, p. 239). On the other hand, two, three and so on are *saṃkhyāpramāna* (number-measure). It is, according to the *Tatvārthavārtika* (Jain, Mahendra 1999, *sūtra* 3.38, p. 206) and the *Trilokasāra* (Mukhtara and Patni 1975, v. 12, p. 13), one of the two types of *dravyamāna*. It has three divisions: *saṃkhyāta* (numerate), *asaṃkhyāta* (innumerate) and *ananta* (infinite). *Asaṃkhyāta* (innumerate) is further divided into three sub-classes: *parita* (preliminary), *yukta* (proper) and *asaṃkhyāta* (innumerate). *Ananta* (infinite) is also divided into three sub-classes: *parita* (preliminary), *yukta* (proper) and *ananta* (infinite). Each of *saṃkhyāta* (numerate), three sub-classes of *asaṃkhyāta* (innumerate) and three sub-classes of *ananta* (infinite) is again divided into *jaghanya* (infimum), *madhyama* (intermediate), and *utkṛṣṭa* (supremum) (Mukhtara and Patni 1975, vv. 13-52, pp. 14-49). This twenty one-fold number system[12] was used by the Jainas to demonstrate their *Karma* theory and cosmology. The other *dravyamāna* is *upamāpramāna* (simile-measure). It is of eight kinds. They are *playa*, *sāgara*, *sūcyaṅgula*, *prataräṅgula*, *ghanāṅgula*, *jagacchreṇī* (or *jagatśreṇī*), *lokapratara* (or *jagatpratara*), and *loka* (Jain, Mahendra 1999, *sūtra* 3.38, pp. 206-208; for details also see Mukhtara and Patni 1975, vv. 12 and 92-112, pp. 13 and 86-109).

This twenty one-fold number system seems to have been said *alaukika gaṇita* by Ṭoḍaramala (1720–1767 CE). Almost one century earlier than him it was called *lokottara giṇatī* (post-worldly reckoning) by Hemarāja in his *Gaṇitasāra* (Jain, Anupam 1988b, v. 4, p. 56). The same seems to have been discussed in the *Ālaukika-gaṇita* ("Non-worldly mathematics") of an anonymous author, of which copy is said to have been preserved in Pañcāyatī Mandira, Delhi (Jain, Anupam 1988a, p. 25). "*Gommaṭasāra grantha meṃ upayogī alaukika gaṇita kī kucha saṃjñāoṃ kā khulāsā* (Eng. Revelation of some terms, applied in the book *Gommaṭasāra*, of non-worldly mathematics)", a write-up inserted just after the foreword (*prāgnivedana*) into the *Gommaṭasāra* (*Karmakāṇḍa*) of Nemicandra, edited by Khubachand Jain, reads that *saṃkhyāpramāna* (number-measure) of twenty one-fold and *upamāpramāna* (simile-measure) of eight kinds along with *kṣetramāna* (space-measure) that contains *pradeśa* (indivisible part of space (*ākāśa*)), *kālamāna* (time-measure) that contains *samaya* (indivisible part of time), and *bhāvamāna* (thought-measure) that contains *avibhāgapraticcheda* (indivisible corresponding-section of omniscience (*kevalajñāna*)) pertain to *alaukika gaṇita* (Jain, Khubchanda 1986/1913, pp. 6-11). On the basis of the above facts, the present author is of the opinion that *saṃkhyāpramāna* (number-measure) is *alaukika gaṇita* if we confine ourselves to *saṃkhyāpramāna* alone, and all of the four *lokottara mānas* (post-worldly measures) are *alaukika gaṇita* if our concern is both *saṃkhyāpramāna* and its area of application.

5. Discussion

5.1. No directive, except the literal meaning of *laukika gaṇita* and *lokottara gaṇita*, has been issued by the Jainas, which shall help us to determine what mathematical thought will be placed in the category of *laukika gaṇita* and what one in the category of *lokottara gaṇita*. This intricacy will get illustrated if the following example is paid attention. The formula, $v \approx \left(\frac{9}{2}\right)\left(\frac{d}{2}\right)^3$, for finding the volume of a sphere whose diameter is d, referred to by Mahāvīra (c. 850 CE) in the *Gaṇita-sāra-saṅgraha* (Padmavathamma, 2000, v. 8.28½, pp. 612-613) is also found in the *Trilokasāra* of Nemicandra (c. 981), that too employed in the process of finding the first *asaṃkhyāta* (innumerate) (Mukhtara and Patni 1975, v. 19 first hemistich, p. 25). Theorization of the school into the two classes does not put any hurdle to accept that the formula is a content of the treatises of the both classes. On the other hand, it is very difficult to explain whether the formula is *laukika* or *lokottara* if the area of its application is taken into consideration.

Laukika gaṇita and *lokottara gaṇita* are the two divisions of mathematics in the school while the canonical class and the exclusive class are the two divisions of the school. The canonical class does not stand for *lokottara gaṇita* although most of the latter are the contents of the treatises belonging to the former. Similar is the case of the exclusive class and *laukika gaṇita*.

5.2. Though the list of the mathematicians of the exclusive class is smaller than that of the canonical

[12] For understanding this system it is suggested to read Singh, Navjyoti 1991, pp. 209-232.

class, the mathematicians contained in it are very important with regard to ancient and medieval Indian mathematics. The reason behind to be the small list is that the Jaina school of Indian mathematics has generally been under the domination of its canonical class. The authors of the exclusive class pay obeisance to those of the canonical class and state that they have gleaned material from the treatises of the canonical class (Padmavathamma 2000, vv. 1.17-1.19, p. 6; v. 1.70, p. 20; v. 7.49, p. 453). This is why the status of the canonical class can be said to be upper than that of the exclusive class in the Jaina school of Indian mathematics.

This claim and others, including the one regarding *laukika gaṇita* and *alaukika gaṇita*, of the present author get support from the following facts revealed and views expressed by Catherine Morice-Singh.

> "During the one hundred and odd years since 1912, much has been written on the *Gaṇitasārasaṅgraha*'s mathematical contents, but no attempt has been undertaken to re-examine the text established by ‹M.› Rangacharya‹, its first editor,› nor to trace the Jaina philosophical and cosmological elements in it, in spite of the fact that Jaina Studies has developed rapidly during the 20th century. The importance given to mathematics (*gaṇita*) by the Jaina thinkers who wanted to quantify in full details the entities existing in the universe is now well known, and the technical and specialized Jaina vocabulary attached to it is also better understood (Morice-Singh 2016, p. 41)."

Here it may be noted that *Mathématiques et cosmologie jaina Nombres et algorithmes dans le Gaṇitasārasaṅgraha et la Tiloyapaṇṇattī* is her doctoral thesis. "In the *Gaṇitasārasaṅgraha*," further writes she, "the exceptionally developed and well-written introductory chapter supplies a great amount of details about the organization of mathematical topics and many explicit references to the Jaina context. The *Gaṇitasārasaṅgraha*'s first two chapters are then both fundamental, and in my thesis I have proposed a deep study of them along with a French translation. In order to re-examine Rangacharya's text and to identify his editorial choices, I examined some manuscripts available at the Government Oriental Manuscripts Library (Madras) hoping to find traces of his work and, in order to get a wider view on the elements of mathematics linked to the Jaina universe, I explored excerpts of different original texts (*Dhavalā*, *Trilokasāra*, etc.) but mainly of the *Tiloyapaṇṇaṭṭī*, a Prakrit text (~ 6th to 9th century) belonging to the same *Digambara* tradition ‹to which the *Gaṇitasārasaṅgraha* belonged›. The study of the impressive mathematical content of these texts has led me to propose answers to the two questions about the *Gaṇitasārasaṅgraha*'s structure … To express numbers, Mahāvīrācārya makes an intensive use of the word-numeral (*bhūta-saṃkhyā*) system, choosing often words belonging to the Jaina terminology, as for example *leśyā*, associated to number 6. Here, Rangacharya, probably not knowing the meaning of *leśyā*, deliberately corrected it into *lekhyā* ‹(v. 2.34)›, which is incorrect … Mahāvīrācārya has, in every aspect of his work, managed to retain the essential and to separate "*alaukikagaṇita*" from "*laukikagaṇita*" without departing from the teachings of his tradition. For instance, the units of length in the *Gaṇitasārasaṅgraha* (‹v.› 1.25) start with the atom (*aṇu*) which is made of an *ananta* quantity of ultimate particles (*paramāṇu*), and an *asaṃkhya* number of *samaya* is required to constitute the first unit of time, the *āvalī* (‹v.› 1.32): The distinction between *ananta* and *asaṃkhyāta* is kept here, even if its utility doesn't appear in a mathematical text (Morice-Singh 2016, pp. 41-43)."

5.3. As far as their chronological order is concerned, the exclusive class must have appeared later than the canonical class. We do not know who all were the mathematicians of the exclusive class prior to Śrīdhara. However, it is certain that this class was in existence long before him.

Following Siddhasena Gaṇi, the commentator of the *Tattvārthādhigama Sūtra Bhāṣya* ("Commentary on the aphorisms of the learning and meaning of the fundamental principles") of Umāsvāti (some period between 150 BCE and 219 CE), B B Datta is, about the mathematical formulae quoted in the *Tattvārthādhigama Sūtra Bhāṣya* by Umāsvāti, of the following opinion. "Umāsvāti's name has come down to us as a great writer on the Jaina doctrines, but not as a writer on mathematics. He is not even known to have ever devoted himself to a study of this science. Hence it will have to be concluded that the mathematical formulae quoted in his *Tattvārthādhigama Sūtra Bhāṣya* were taken from some other treatise on mathematics known at his time (Datta 1929, pp. 126-127)." The time when it happened seems to be the one before which the exclusive class of the school began to come forward. The period preceding the fifth century CE or preceding the time of Āryabhaṭa I (born 476) is considered to be the darkest period of the history of Indian mathematics (Singh, A. N. 1942, p. 4). It was that period during which the exclusive class seems to have been struggling for its executive shape and it finally came in that shape before or by Śrīdhara's time. This is why we do not find measure to have been classified into *laukika* and *lokottara* in the *Anuyogadvāra Sūtra* (3rd century CE) (Madhukara 1987, *sūtra*, 327, p. 239).

5.4. The exclusive class seems to have helped the society a lot by producing the treatises exclusively written on mathematics as and when its mathematician got the seat in the court of the state. Mahāvīra seems to have worked at the court of the famous and benevolent ninth-century *Rāṣṭrakūṭa* king Amoghavarca Nṛpatuṅga who ruled at Mānyakheta in south India, much of what is known as Karnataka today as he has praised the king in glowing terms and wished for his prosperity in the *Gaṇita-sāra-saṅgraha* (Padmavathamma, 2000, vv. 1.3-1.8, pp. 2-3 and Plofker, 2009a, p. 162). Rājāditya flourished either around 1120 CE in the royal court of the king Viṣṇuvardhana, who reigned from 1111 CE to 1141 CE, of the Hoyasal dynasty (Padmavathamma et al, 2013, pp. xxiii-xxiv) or around 1190 CE in the court of the king Varaballāḷa II, who ruled from 1173 CE to 1220 CE, whom he referred to as Viṣṇunṛpāla (Padmavathamma et al., 2013, pp. xxiv-xxv). Ṭhakkara Pherū held the positions during the period of the successive Sultans Alauddin Khaljī (1296–1316 CE), Shihabuddin Umar (1316 CE), Qutubddin Mubarak Shah (1316–1320 CE) and Ghiyasuddin Tughluq (1320–1325 CE) at their treasuries (SaKHYa 2009, p. xiii).

5.5. It may need a separate paper to list and discuss the achievements of the Jaina school of Indian mathematics. Some remarkable achievements of the school, which distinguish the school from other ancient schools, R C Gupta has summarized as follows.

> "Closed and open number systems both finite and transfinite were developed. The Jainas had realized the notion of actual infinity in the realm of numbers, formulated the idea of cardinality, and thus made first attempts towards the calculus of transfinite numbers. Logarithms (especially to base two) were applied and their laws of combinations were made known. Mathematics of transfinite class (called *ananta*) was dealt. In fact, the mathematical operations developed to handle transfinite numbers, was one of the greatest achievements of the Jainas. The Jaina *Karma* system has been developed, like modern system theory, on the basis of several postulates and hypothesis, and utilizing such notions as that of one-to-one correspondence. Ideas of structuralism and functionalism of system theory have been developed. System-theoretic knowledge of maxima and minima was evolved. Several set-theoretic relations are found quoted in Prakrit texts. Fourteen special divergent sequences have been discussed. … Ten

types of infinities are mentioned in canonical texts (Gupta 1993, p. 24)".

These achievements are, according to R C Gupta, from ancient *lokottara gaṇita* of the school (Gupta 1993, p. 24). The present author too would like to point out that these achievements belonged to only the canonical class of the school. The exclusive class, which separated from the canonical class to provide mathematics education to common people, never referred to these achievements; perhaps for the reason that it could not find the areas of their application in the contemporary civil life.

5.6. We have noticed, although in brief, above that the Jaina school of Indian mathematics lasted for more than two thousand years. It not only prolonged but also it was varied in terms of using the languages, although Prakrit along with its different forms was the most used one among them, and having different purposes including the demonstration of canonical thoughts using mathematics and providing mathematics education to the civil society. The division, whatever it may be, of the school of this sort cannot be an ideal one. This is illustrated from the following examples.

Mādhavacandra Traividya was an immediate disciple of Nemicandra as he himself claims to be so (Mukhtara and Patni 1975, v. 1, p. 768). He wrote a very useful commentary on the *Trilokasāra* of Nemicandra (981 CE). The most interesting, especially for the historians of mathematics, feature about his commentary is that it contains rationales to mathematical rules given in the *Trilokasāra*. He is said to have authored the *Ṣaṭtriṃśikā* ("‹The textbook› of thirty six ‹logistics and determinations›"). It is the refined-essence (*śodhya-sāra*) of the ‹*Gaṇita*›*-sāra-saṅgraha* of ‹Mahā›Vīrācārya (Jain, Anupam 1988c, pp. 65-72). Since his commentary contains mathematics along with canonical discussion while the *Ṣaṭtriṃśikā* is exclusively on mathematics, he must belong to both of our theorized classes.[13] It is hereby decided that a particular mathematician can be placed in both of the classes if his one treatise belongs to the canonical class and the other belongs to the exclusive class but one particular treatise of any mathematician should not be placed in both of them.

The *Gaṇitasāra* was composed, in eighty eight Hindi verses, by Hemarāja (c. 1673) exclusively on the twenty one-fold number system. It seems to have been aimed at imparting education of that system to common Jaina Śrāvakas (devout listeners) of his time (Jain, Anupam 1988b, 56). This is why the present author suggests placing it in the exclusive class although it contains *lokottara gaṇita*.

There are also many such treatises that contain mathematical thoughts not along with canonical discussion nor are written exclusively on mathematics. Example of this sort is the *Chando'nuśāsana* of Hemacandra (1088-1172 CE). He was a scholar of Jaina faith. The treatise is on Prakrit prosody. It contains mathematical thoughts pertaining to combinatorics (Alsdorf 1991, pp. 20-31). Since it is independent from the canonical discussion, we suggest placing it in the exclusive class. If not, for it, possibly a miscellaneous class has to be devised.

5.7. The way in which mathematics dealt and developed by the Jainas was much more than a tradition. It was a school, which we have noticed from the beginning of this paper. It was not closely related to the mainstream "classical Sanskrit mathematical writings", as far as language and mathematics are concerned, as most of its treatises were composed in Prakrit and most of the mathematics dealt and developed in the canonical class is entirely different. At the same time, it seems to be closely related to the mainstream

[13] Earlier to this paper Mādhavacandra Traividya was placed in the exclusive class alone by the present author. See Jadhav 2013, pp. 46 and 48. This paper onwards, he changes his view in this regard.

"classical Sanskrit mathematical writings" only if a few treatises, for example, those of Śrīdhara and Mahāvīra, of the exclusive class are assessed at face value. Śrīdhara was the most distinguished mathematician of his time. His reputation spread all over India. Similarly, Mahāvīra was a celebrated mathematician of his time. His fame rests on the *Gaṇita-sāra-saṅgraha*. It was used as a text-book for centuries in the whole of south India. Their treatises being composed in Sanskrit, they appear to be members of the mainstream "classical Sanskrit mathematical tradition" but when we go in detail we find that the case is not so. For example, trapezium, especially isosceles trapezium, was a household geometrical figure for the Jainas. Each of the front and backside faces of the three fold universe of the Jainas is in this shape. Śrīdhara gave it so importance that he considered it primary figure.[14] In Jaina cosmography, the middle universe is supposed to be a flat plane divided into an innumerable number of concentric annuli which are alternatively islands and seas. Following this concept, Mahāvīra coined the expressions *bahiścakravālavṛtta* (outer-annulus-circle i. e., the outer circle of an annulus) and *antaścakravālavṛtta* (inner-annulus-circle i. e., the inner circle of an annulus) (Padmavathamma, 2000, v. 7.6, p. 427; Jadhav, 2013, pp. 97-98 and 558).

The expressions such as "Sanskrit mathematics and astronomy" (Plofker, 2010, p. 1), "Sanskrit mathematicians" (Høyrup, 2012, p. 2), "Sanskrit formulas" (Plofker, 2001, p. 284), "Sanskrit mental-calculation algorithms" (Plofker 2009a, p. 16), "Sanskrit geometry" (Plofker, 2009a, p. 28) and so forth are mostly popular in foreign publications on the history of Indian mathematics. Why these sorts of expressions are particularly followed in those publications is not known. However, "Sanskrit mathematics" refers to mathematics contained in the treatise composed in Sanskrit. "Sanskrit mathematicians" mean to be those mathematicians whose treatises are in Sanskrit. In the same manner we shall have to interpret the remaining expressions. Sometimes we come across the expressions such as "Prakrit mathematical work" (Plofker 2009a, p. 209). Indian researchers, if not at all, rarely employ them. Those expressions, consciously or unconsciously, intend to show a linguistic division of ancient and medieval Indian mathematics, especially to a common reader. In fact, original mathematical thoughts were developed in linguistically varied India[15] irrespective of language although Sanskrit has been the pan-Indian medium of intellectual discourse. In the section 5.5 of this paper, we have already noticed the highly original mathematical thoughts of the Jainas. All of them, belonging to the canonical class of their school, were composed in Prakrit. Most of them never found any place in the treatises composed in Sanskrit. A few of them had found some place in some Sanskrit texts such as in Mādhavacandra Traividya's Sanskrit commentary on the *Trilokasāra* of Nemicandra, but they were not paid any attention by the intelligentsia of the perceived "classical Sanskrit mathematical tradition". For example, the two important mathematical concepts *addhached* (Skt. *ardhaccheda*, logarithms to the base two) (Jadhav 2002a; 2003; 2014) and *vaggidasaṃvaggida* of *a* (Skt. *vargitasaṃvargita* of *a*, 'the self-power of *a*' or 'raising *a* to its own power' where *a* is a positive integer) (Jadhav, 2008) always remained untouched by the others including the exclusive class.

[14] Dvivedi 1899, vv. 42-43 and examples vv. 80-81, pp. 30-32; Shukla 1959, v. 115, p. 161; examples vv. 122-124, pp. 161-162; vv. 126-127, p. 165; Jadhav 2013, pp. 157-160 and 558.

[15] That how poor is the status of the mapping of the mathematical literature composed in the ancient and medieval Indian regional languages can be had from: Sarma 2011, pp. 201-211; SaKHYa 2009, p. xi.

6. Concluding remarks

The Jaina school of Indian mathematics is an approved school in the field of history of ancient and medieval Indian mathematics. The division of the school into the canonical class and the exclusive is based on theorization. The approaches adopted for theorization can be seen in the sections three and five; the former initiates its process while the latter establishes it by means of analysis.

One should not try to form the image of the school from the works of its exclusive class alone. The image which lies with its exclusive class is the outer one. The intrinsic image of the school lies with its canonical class.

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